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Research Article

Comprehensive Performance Analysis of Non Blind LMS Beamforming Algorithm using a Prefilter

Swapnil M Hirikude[A*] and A.N.Jadhav[B]

[A]Department of E & TC,SSDGCT's Sanjay Ghodawat Group of Institutions,Shivaji University, Kolhapur,Maharashtra,India.416118
[B]Department of E & TC, D.Y.Patil College of Engg. & Tech., Shivaji University, Kolhapur, Maharashtra,India.416006

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Abstract

The demand for wireless mobile communications services is growing at an explosive rate, the high demand for wireless communication services in 3G and now 4G need more system capacity. The most elementary solution would be to increase bandwidth; however, this becomes even more challenging as the electromagnetic spectrum is becoming increasingly congested. The frequency reuse concept increases capacity however, increasing the number of cells to accommodate growing subscriber needs is not effective and not an economical option. This has led to development of new technologies that exploit space selectivity. This is done through smart-antenna arrays and the associated adaptive beam forming algorithms. In reality, antennas are not smart; it is the digital signal processing, along with the antenna, which makes the system smart. When smart antenna with Adaptive beamforming is deployed in mobile communication using either time division multiple access (TDMA) or code division multiple access (CDMA) environment, exploiting time slot or assigning different codes to different users respectively, it radiates beam towards desired users only. Each beam becomes a channel, thus avoiding interference in a cell. Smart-antenna systems provide opportunities for higher system capacity and improved quality of service. A new beamforming (Hybrid) technique using a pre-filtering process that decreases noise and interference effects to improve performance of cellular systems is illustrated here. This paper presents e comprehensive analysis ththe results obtained by applying prefiltering process to the most researched LMS non blind beam forming algorithm.

Keywords: *Smart Antenna, DOA, Beamforming, Prefilter, LMS.*

1. Introduction

Adaptive beamforming can be classified into two categories: Non-blind adaptive algorithms and blind adaptive algorithms (Lal.C.Godara *et al*, July 1997; Lal.C.Godara *et al,* August 1997). Non-blind adaptive algorithms need statistical knowledge of the transmitted signal to converge to a solution. This is typically accomplished through the use of a pilot training sequence sent over the channel to the receiver to help it identify the desired user. On the other hand, blind adaptive algorithms do not need any training; hence the term „blind‟ is used. They attempt to restore some characteristics of the transmitted signal in order to separate it from other users in the surrounding environment. After the detailed study of existing beamforming algorithms and their applications detailed in (Lal.C.Godara *et al*, July 1997; Lal.C.Godara *et al,* August 1997; J.C.Liberti *et al* 1999) it is seen that there is still room to improve the performance of conventional beamforming algorithms. In this paper a prefiltering technique proposed in [3] which is used with the non-blind algorithms to enhance their performance is presented. This technique acts on the input signal vector **x** (k) as a band pass filter but in spatial domain, so it minimizes the noise and interference effects as a function of the Direction of Arrival (DOA).

2. The Prefiltering Technique

The proposed prefiltering technique aims to increase the Signal-to-Interference and Noise Ratio (SINR) of the beam forming system by reducing the interference and noise effects on the desired user signal using filtering in spatial domain, or extracting the desired signal from the instantaneous input signal vector x(k) of the beamformer, (Abu-Ella O *et al* ,2008; Abu-Ella O *et al* , 2010) as can be seen in Fig. 1.In this context, it is worth pointing out that in image processing, especially in image compressing techniques, one can find an abundance of techniques that can reconstruct the original image with acceptable performance, without using all transformation components, but rather using only the lower component coefficients of the image transform matrix (Wintz. P *et al* 1972). This fact is exploited here and employed with some modification in the antenna array processing to obtain a new hybrid beamforming technique.

Since the interfering signals are in the same frequency band of the desired signal, they are analyzed representing

*Corresponsing author: **Swapnil M Hirikude**

them in another domain other than frequency domain so as to distinguish between the mixed signals that form the input signal. Therefore, the technique is based on the idea that the desired and interfering signals arrive at the antenna array from different directions. Thus, these differences between arriving signals can be exploited.

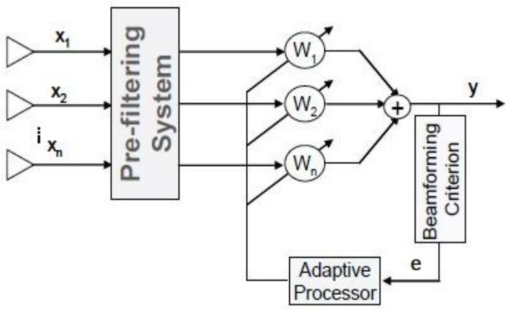


Fig.1 Adaptive beam former with Pre-filtering system

The distinction is obtained by converting the input signal to the spectrum of the spatial domain (this domain is the sine of the direction of arrival, or sinθ domain).The desired signal is extracted from the input signals simply by making a band-pass filter in the spectrum of the spatial domain, i.e. in the sinθ spectrum. This filtering process is shown in Fig. 2, and is explained in (Abu-Ella O *et al* ,2008; Abu-Ella O *et al* , 2010) as follows:

- The Most Significant Coefficient (MSC) of the transformed signal is selected. This is ranked as the largest sample of the transformed desired signal.
- The most significant coefficient is placed at its rank in the M zeros element vector (zero padding).
- The Inverse Fast Fourier Transform (IFFT) is applied to the filtered vector of the previous step to reconstruct an alternative input signal that contains a reduced amount of interference and noise.
- The reconstructed data vector is used as input signal to the conventional adaptive beam forming system.

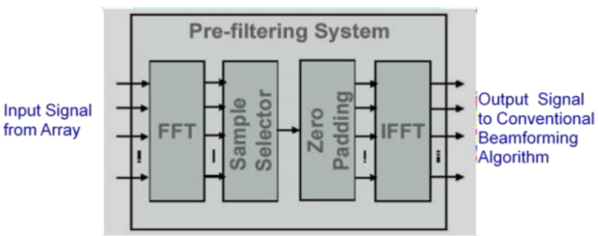


Fig.2 Pre-filtering Process

Mathematically, assuming that the propagation vector for the θ direction of arrival, is given by

$$a(\theta) = e^{-j\,2\pi \frac{d}{\lambda} \sin\theta (m-1)}, m = 1,2,\ldots\ldots, M \qquad (1)$$

Where *M* is the number of array elements, *d* is the spacing distance between any two adjacent elements, and λ is the wavelength of the operating carrier frequency. Applying the FFT on the array propagation (Equation 1) gives,

$$A(\Theta) = \mathrm{FFT}\left(a(\theta)\right)$$
$$= \frac{1}{M}\sum_{m=1}^{M} e^{-j\,2\pi \frac{d}{\lambda}\sin\theta(m-1)} e^{-j\,2\pi K(m-1)/M}$$
$$= \frac{1}{M}\sum_{m=1}^{M} e^{-j\,2\pi\left(\frac{d}{\lambda}\sin\theta(m-1)+\frac{K}{M}(m-1)\right)} \qquad (2)$$

Assuming d= λ/2, and solving Equation (2) and equating the result to zero, the following formula gives the index KMSC (or the order) of the most significant coefficient as a function of the direction of arrival θ and the number of array elements M.As KMSC.must be an integer the equation takes the form of equation (3).

$$K_{\mathrm{MSC}} = \mathrm{mod}_M \left\{1 + M\left[-\frac{1}{2}\left(\frac{M(2\pi M \sin\theta - \pi \sin\theta)}{\pi(2M-1)}\right)\right]\mathrm{int}\right\} \qquad (3)$$

Where mod M is the modulus notation performed on M points. Equation (3) can be simplified to

$$K_{\mathrm{MSC}} = \mathrm{mod}_M\left(\left[1 - \frac{1}{2}\ M \sin\theta\right]\right) \qquad (4)$$

This Kmsc is used then to reconstruct the modified input signal which has reduced interference and noise. Simulation results presented later in this paper show that the prefiltering technique significantly reduces the mean square (MSE). This prefiltered output is then used as input to any conventional beamforming algorithm to enhance its performance. The results obtained by applying this technique to the non blind Conventional LMS algorithm are discussed here.

3. LMS Algorithm

In adaptive filtering applications for modeling, equalization, control, echo cancellation, and Beam forming, the widely used least-mean-square (LMS) algorithm has proven to be both a robust and easily-implemented method for on-line estimation of Time-varying system parameters (S. C. Douglas *et al*, *1994*).

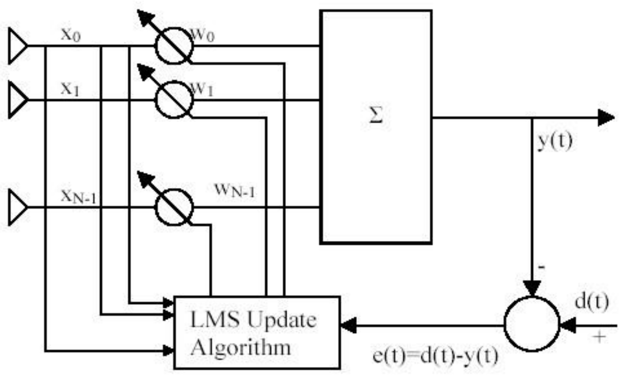


Fig.3 LMS adaptive beamforming process

Fig.3 shows a generic adaptive beamforming system which requires a reference signal. As shown in Fig.3, the outputs of the individual sensors are linearly combined after being scaled using corresponding weights such that the antenna array pattern is optimized to have maximum possible gain in the direction of the desired signal and nulls in direction of interferers (Lal.C.Godara *et al*, 1997; S. C. Douglas *et al, 1994;* R. S. Kawitkar *et al* 2005*)*.

LMS is nonblind algorithm which requires a training sequence of known symbols $d(n)$, to train the adaptive weights. It uses the estimate of the gradient vector from the available data. This algorithm makes successive corrections to the weight vector in the direction of the negative of the gradient vector which finally concludes to minimum MSE (MMSE).

This successive correction to the weight vector is the point at which optimum value w0 is obtained that relies on autocorrelation matrix R and cross correlation matrix p of the filter. LMS is an adaptive beamforming algorithm, defined by the following equations (Lal.C.Godara *et al*, 1997; S. C. Douglas *et al, 1994;* R. S. Kawitkar *et al*, 2005 ; B.Widrow *et al*, 2005;Simon Haykin *et al*, 2002) with input signal $x(n)$:

$$y(n) = W^H(n)\, x(n) \quad (5)$$

$$e(n) = d(n) - y(n) \quad (6)$$

$$w(n+1) = w(n) + \mu\, x(n)\, e^*(n) \quad (7)$$

where y (n) is the filter output,e(n) is the error signal between filter output and desired signal d(n) at step n. d(n) is the training sequence of known symbols (also called as a pilot signal), required to train the adaptive weights. Equation (7) is the weight w(n) update function for the LMS algorithm. μ is rate of adaption also called as a step size, controlled by the processing gain of the antenna. (R. S. Kawitkar *et al*, 2005*)* ; B.Widrow *et al*, 2005).

4. Hybrid (Prefiltered) Adaptive Beamforming Algorithm

The complete hybrid system by applying prefiltering to the conventional adaptive beamforming algorithm discussed in sections II and III respectively is illustrated below with the help of flowchart in Fig 4. The technique aims at improving the performance of beam forming algorithm by reducing the interference and noise effects on the desired user signal.

5. Simulation Results

Simulation of the technique is carried out using MATLAB software. The prefiltered signal is applied to the conventional LMS beam forming algorithm for a Uniform Linear array (ULA) with a distance between the elements d = λ/2. Results of magnitude response of Conventional and Hybrid LMS beamforming algorithms presented here are obtained by varying parameters like no of antenna elements (M) and step size parameter μ, for two or more interferes in random directions and Noise is assumed to be Gaussian.

The performance analysis is done using following parameters such as Beam Pattern gain characteristics, Signal to Interference Ratio SIR with respect to number of iterations, Bit Error rate, Convergence speed and system capacity i.e number of users that can be served by the system.

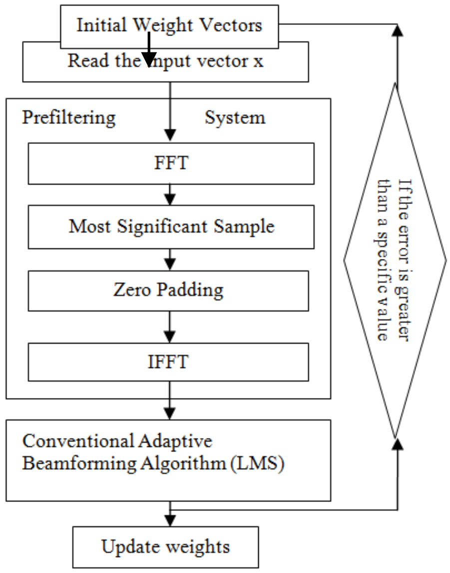


Fig.4 Flowchart of the Hybrid (Prefiltered) adaptive beamforming technique

Fig.5 a) Shows the beam pattern gain (magnitude response) of Conventional and Hybrid LMS algorithm for M =8, desired angle at=45 deg interference angles at 35, 50. μ=0.001 Fig.5 b) & c) show the Polar plot for the same in terms of the Antenna Array factor.

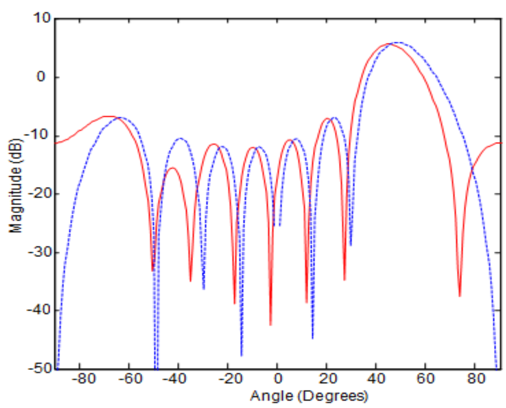


Fig.5 a) Beam pattern gain of LMS and hybrid technique for M=8, μ=0.001

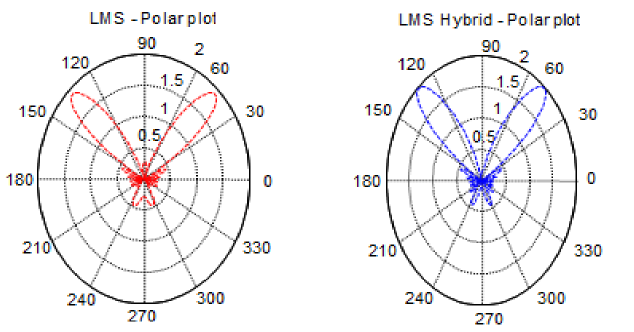


Fig.5 b) Polar plot for LMS c) Polar plot for Hybrid LMS

Fig.6 a) Shows the beam pattern gain (magnitude response) of Conventional and Hybrid LMS algorithm forM =8, desired angle at=45 deg interference angles at 35, 50. μ=0.001 Fig.6 b) & c) show the Polar plot for the same in terms of the Antenna Array factor.

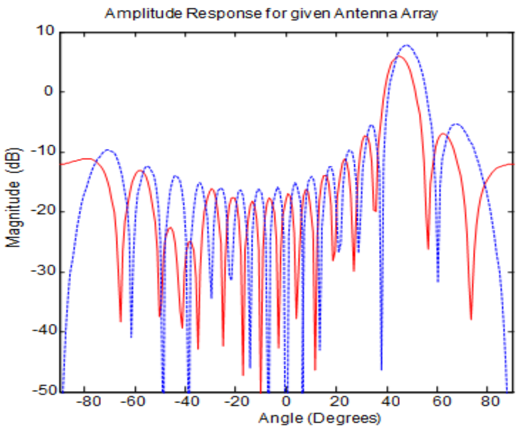
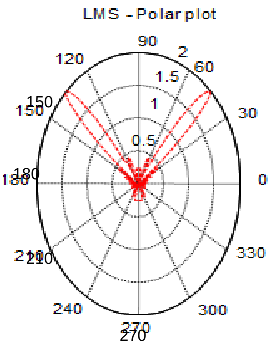


Fig.6 a) Beam pattern gain of LMS and hybrid technique M=16, and μ=0.001



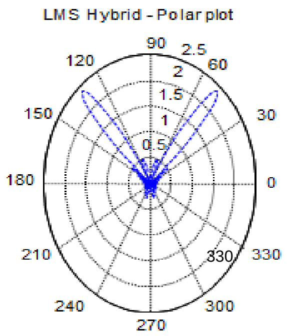


Fig.6 b) Polar plot LMS c) Polar plot Hybrid LMS

Plots obtained here do not give exact value of amplitude /gain response G (θ), hence for more accurate estimation we normally refer its computed value. Hence for the further analysis we refer its computed value in MATLAB.

After observing Table 1, 2 & 3 show that the prefiltered technique improves antenna beam pattern gain than the conventional LMS algorithm for most of the DOA's. The technique works well even for close angular separation between desired user and interferers when the antenna elements are increased.

Table 1. Results obtained for Number of antenna elements M=8 and μ=0.001.

Input DOA (Θ) deg	Beam Gain for Conventional LMS in (db)	Beam gain for Prefiltered LMS in (db)	Total improvement Beam Gain (db)
10	5.5956	6.9192	1.3236
30	5.59	5.65	0.06
45	5.65	5.8	0.15
60	5.4486	7	1.5514
90	5.62	5.41	-0.21
120	5.48	6.96	1.48

Table 2. Results obtained for Number of antenna elements M=12 and μ=0.001

Input DOA (Θ) in deg	Beam Gain for Conventional LMS in (db)	Beam Gain for Prefiltered LMS in (db)	Total improvement Beam Gain (db)
10	5.94	8.09	2.15
30	5.8737	5.9386	0.0649
45	5.9406	6.5231	0.5825
60	5.9417	6.3147	0.373
90	5.9	5.54	-0.36
120	5.9284	6.205	0.2766

Table 3. Results obtained for Number of antenna elements M=16 and μ=0.001

Input DOA (Θ) in deg	Beam Gain for Conventional LMS in (db)	Beam Gain for Prefiltered LMS in (db)	Total improvement Beam Gain (db)
10	5.9955	8.9414	2.9459
30	5.95	5.98	0.03
45	6.0068	7.5838	1.577
60	5.9787	5.9942	0.0155
90	5.9977	5.9016	-0.0961
120	5.9961	6.1109	0.1148

The Second parameter for comparison is Signal to interference Ratio SIR behavior with respect to number of iterations. For the same initial conditions set for the previous case graphs are plotted for LMS and Prefiltered (PF) LMS by increasing the number of iterations for DOA 45 deg & M=8. It is seen from the Fig.7 a) & b) that there is an improvement of 0 to 3 dbs in the SIR of the prefiltered algorithm than the conventional algorithm and also the improvement is achieved in the initial few iterations only and becomes steady with number of iterations from 200 to 800.The simulation is also carried out by varying the number of antenna elements 8, 12 and 16 for constant number of iterations which also shows improvement in SIR form 0 to 3dbs.
0 to 3dbs.

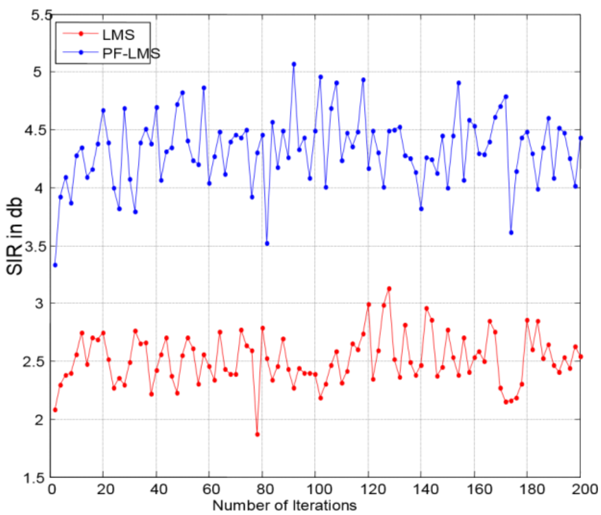


Fig.7 a) SIR versus the number of iterations Number of

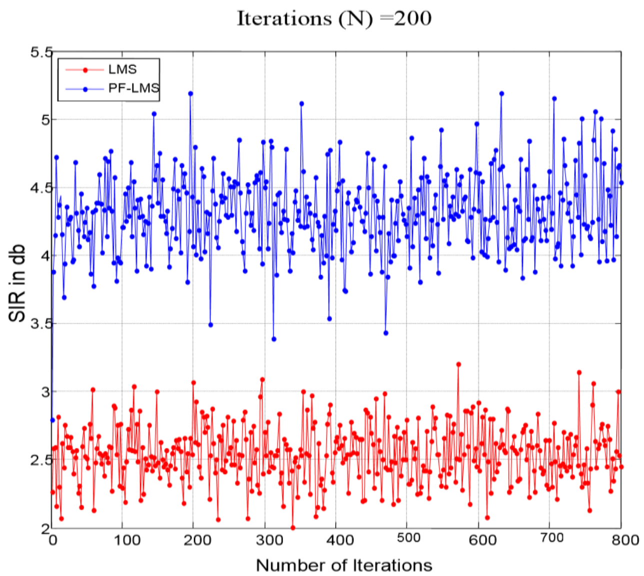


Fig.7 b) SIR versus the number of iterations Number of Iterations (N) = 800

The Third Parameter for the performance analysis is the Bit error rate for varied SNR. Fig.8 a) & b) shows the behavior of BER when the SINR is varied from -20 db to +20db for DOA 45º and antenna element M= 4 &12 respectively. It can be seen that BER of the proposed Prefiltered algorithm matches the Conventional algorithm and improves with the increase in SINR. Table 4 shows BER behavior for different DOA‟s and Table 5 shows Difference in BER when antenna elements are increased to 8, 12 and 16. It can be seen that BER of the proposed Prefiltered algorithm matches the Conventional algorithm and improves with the increase in SINR. It also indicates that for some DOA‟s conventional algorithm performs better while for certain DOA‟s Prefiltered algorithm performs better. Difference between the minimum BER achieved by Conventional and Prefiltered (Hybrid) algorithm is very less.

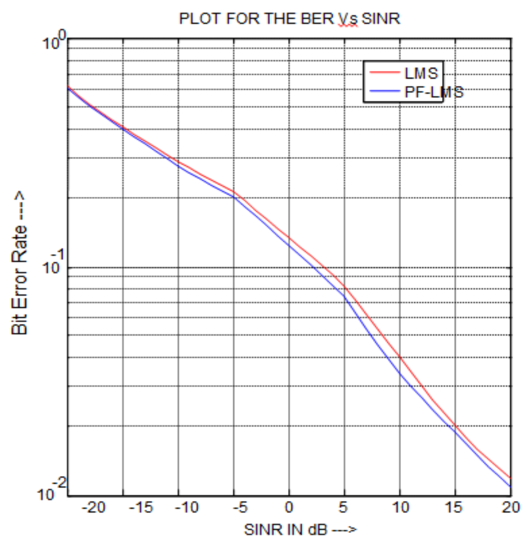


Fig.8.a) Plot for the BER Vs SINR for Antenna Elements (M) = 4

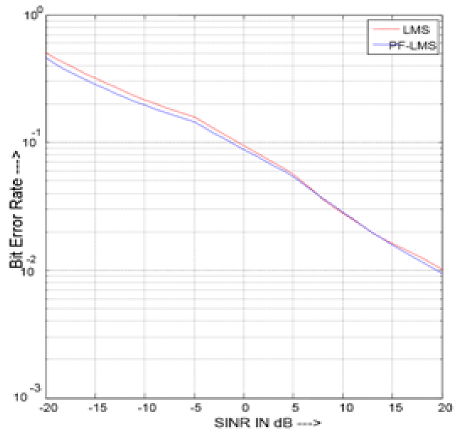


Fig.8.a) Plot for the BER Vs SINR for Antenna Elements (M) =12

Table 4. Comparison of minimum BER for different DOA's and With SINR variation from -20 dB to 20 dBs. & M=8

DOA in (deg)	Minimum BER using LMS	Minimum BER using Prefiltered LMS	Difference In BER
10	0.0117	0.011	-0.0007
30	0.0115	0.011	-0.0005
45	0.0136	0.0136	0.000
60	0.0117	0.013	0.0013
90	0.0097	0.0137	0.004
120	0.0117	0.0137	0.002

Table 5. Comparison of minimum BER for different DOA's and antenna elements M=8, 12 &16 respectively.

DOA in (deg)	Difference in BER with M=8	Difference in BER with M=12	Difference in BER with M=16
10	-0.0007	-0.0005	0.0007
30	-0.0005	0.0007	0.0011
45	0.000	0.0007	-0.0011
60	0.0013	-0.0011	0
90	0.004	0.0022	0.0013
120	0.002	-0.0011	0

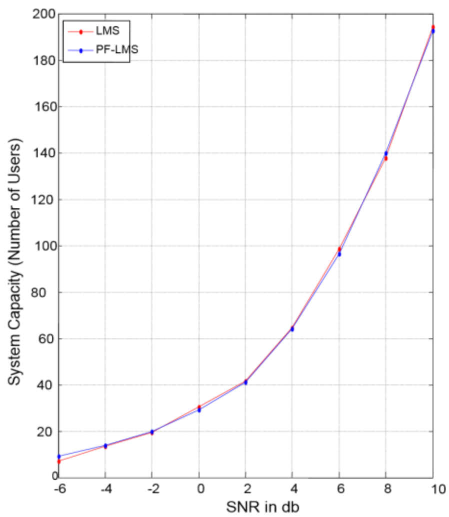


Fig.9 a)System Capacity (Number of users) plot for DOA 45º, M= 4 and SNR Variation from -6 to 10 dbs

The Next parameter for Comparison is the Capacity i.e number of users that can be served by the system with respect to SNR and for different bit rates.

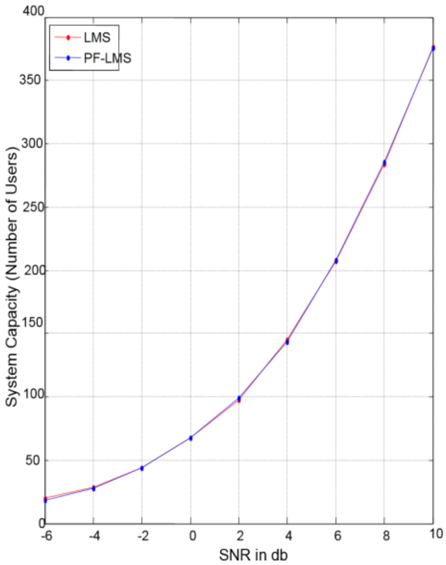


Fig.9 b) System Capacity (Number of users) plot for DOA 45º, M= 16 and SNR Variation from -6 to 10 dbs

Table 6. Comparison of Capacity (Number of Users) for different DOA‟s and With SINR variation from -20 dB to 20 dBs & M=8

DOA in (deg)	Number of Users using LMS	Number of Users using PF- LMS	Difference in Number of Users
10	179	179	0
30	179	178	-1
45	180	179	-1
60	178	179	1
90	177	179	2
120	177	179	2

Table 7. Comparison of Capacity (Number of Users) for different DOA‟s and antenna elements M=8, 12 &16 resp.

DOA in (deg)	Difference in no.of Users with M=8	Difference in no.of Users with M=12	Difference in no.of Users with M=16
10	0	-2	1
30	-1	0	1
45	-1	2	0
60	1	-1	0
90	2	0	1
120	2	1	0

From Fig. 9 a) & b) it is observed that the number of user curves of both the systems almost overlap each other. The number of users increased with the increase in number of antenna elements. Also from Tables 6 & 7 and Tables 8 & 9 is clear that the system capacity increases with the increase in bit rate as well as SINR and the rate of increase in both conventional and prefiltered algorithm is identical.

Table 8. Comparison of Capacity (Number of Users) for different bit rates and antenna elements M=8

Bit Rate (bits/sec)	Number of Users using LMS	Number of Users using PF- LMS	Difference in Number of Users
5000	179	179	0
8000	284	286	2
12000	430	432	2
18000	638	640	2
32000	1140	1142	2

Table 9. Comparison of Capacity (Num.of Users) for different bit rates and antenna elements M=8, 12 &16 resp.

Bit Rate (bits/sec)	Difference in No. of users with M=8	Difference in No. of users with M=12	Difference in No. of users with M=16
5000	0	1	1
8000	2	2	1
12000	2	1	1
18000	2	1	5
32000	2	1	2

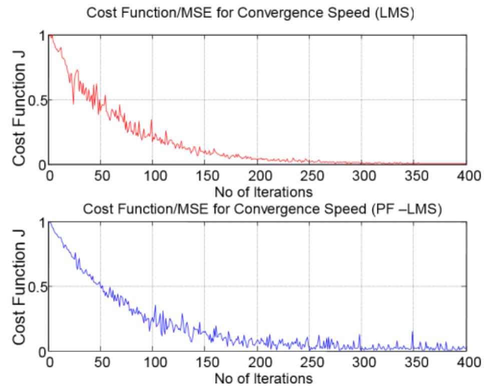


Fig.10 a) Convergence Speed for M= 8, µ=0.001

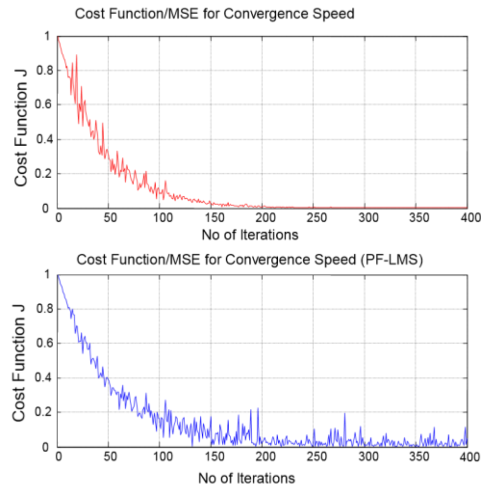


Fig.10 b) Convergence Speed for M= 12, µ=0.001

The Final parameter of comparison is the *convergence speed* determined by measuring the error behavior of the algorithms versus the number of iterations. i.e. measuring the value of the cost function (the mean square error) at each sample time. Fig.10 a) to d) show Comparison of Convergence speed of LMS and Prefiltered (Hybrid) LMS in terms of number of iterations for number of antenna elements M= 8, 12, 16 & 32 & μ= 0.001respectively.

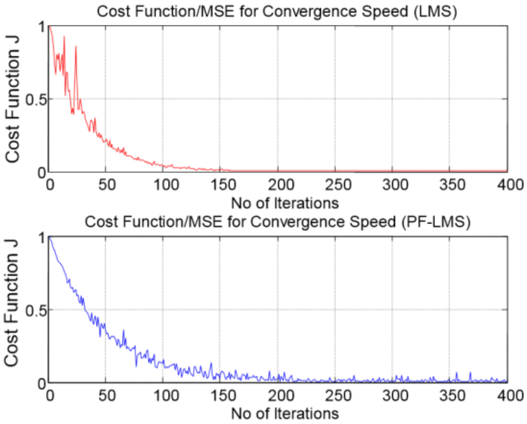


Fig.10 c) Convergence Speed for M= 16, μ=0.001

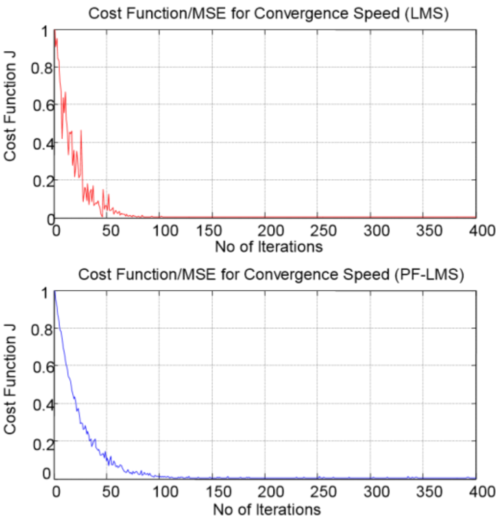


Fig.10 d) Convergence Speed for M= 32, μ=0.001

As the graphs do not show the exact values, the approximate numbers of iterations for different angles of arrivals are tabulated in Tables 10 to 13 for number of antenna elements 8,12,16 32 respectively. With μ=0.001.

Table 10. Convergence Speed for different angles of arrivals and number of antenna elements 8 & μ=0.001

DOA in (deg)	Num. of iterations for LMS	Num. of iterations PF-LMS
10	300	400
30	250	250
45	250	300
60	250	350
80	250	350
90	250	300
120	250	400
150	250	250

Table 11. Convergence Speed for different angles of arrivals and number of antenna elements 12 & μ=0.001

DOA in (deg)	Num. of iterations for LMS	Num. of iterations Hybrid
10	150	250
30	150	250
45	150	250
60	150	250
80	150	350
90	150	200
120	150	200
150	150	200

Table 12. Convergence Speed for different angles of arrivals and number of antenna elements 16 & μ=0.001

DOA in (deg)	Num. of iterations for LMS	Num. of iterations PF-LMS
10	70	100
30	60	80
45	70	100
60	60	80
80	60	100
90	60	100
120	60	100
150	60	100

Table 13. Convergence Speed for different angles of arrivals and number of antenna elements 32 & μ=0.001

DOA in (deg)	Num. of iterations for LMS	Num. of iterations PF-LMS
10	100	250
30	120	150
45	150	200
60	150	250
80	150	200
90	150	200
120	150	200
150	150	200

From Fig 10.a) to d) and Tables 10 to 13 it can be seen that the convergence speed of the conventional algorithm is slightly faster than the Prefilterd technique, but the difference is very small. Both the algorithms converge at a faster rate with increase in number of antenna element. The difference between the numbers of iterations required to converge between Conventional and Hybrid algorithm reduces with increase in number of antenna elements.

6. Discussion and Conclusion

The simulation results obtained for non blind LMS Beamforming algorithm by applying prefiltering technique are presented and discussed in the previous Chapter. These results obtained for the five performance measures can be observed and analyzed to conclude the following Points:

1) Beam Pattern Characteristics:

The beam pattern response of the prefiltered (hybrid) algorithm is better than the conventional algorithm. The improvement varies between 0 to 3 dBs for given experimental conditions. The amplitude response (beam pattern gain) of the hybrid technique for certain angles increases with increase in number of antenna elements even for the close spatial separation between desired user and interferers.•The Spatial accuracy of the Prefiltered (Hybrid) algorithm is slightly less than the conventional algorithm .The difference varies from 0 to 5 degrees. This is because the Prefiltering is done using FFT and then IFFT which affects the spatial accuracy but this improves with increase in number of antenna elements.

2) Signal to Interference Ration (SIR) Vs Number of iterations:

The signal to interference ratio improves by about 1 to 3 dB‟s in the Prefiltered algorithm than the conventional algorithm as the number of antenna elements are increased.The SIR improvement is achieved in the less number of iterations and stays constant as the number of iterations are increased.

3) Bit Error Rate (BER) with respect to SINR:

The BER of the proposed Prefiltered algorithm matches the Conventional algorithm and improves with the number of antenna elements. For certain DOA‟s Prefiltered algorithm performs better than the conventional algorithm. Difference between the minimum BER achieved by Conventional and Prefiltered (Hybrid) algorithms is very less.

4) System Capacity (Number of Users in the system) Vs SNR:

It is observed that the number of users serviced by both the systems is almost same. The number of users increased with the increase in number of antenna elements. Also system capacity increases with the increase in bit rate as well as SINR and the rate of increase in both conventional and prefiltered algorithm is identical.

5) Convergence Speed i.e. Mean Square Error with respect to Number of Iterations:

It can be inferred that the Prefiltered (Hybrid) algorithm is slow to converge as compared to the conventional but the difference is marginal. Both the algorithms converge at a faster rate with increase in number of antenna element. The numbers of iterations are reduced from maximum 250 to minimum 20 as number of antenna elements are increased from 4 to 32 for given experimental conditions. Moreover the difference between the numbers of iterations required to converge between Conventional and Hybrid algorithm reduce with increase in number of antenna elements.

This analysis indicates that Prefiltering technique will be useful to enhance the performance of the Beamforming in systems which are corrupted with noise and interference and significantly especially when there are more number of antenna elements and convergence speed is not of much concern.

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