International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 - 5161

Available at http://inpressco.com/category/ijcet

Research Article

# Estimating Reliability of Degradable Computing System using Fuzzy Logic

Kalika Patrai[Ȧ*] and Indu Uprety[Ḃ]

[Ȧ]Department of Computer Applications, Galgotia Institute of Management and Technology, Greater Noida-201306, U.P., India
[2]School of Management, Gautam Budhha University, Greater Noida-201306, U.P., India

Accepted 01 May 2014, Available online 02 June 2014, **Vol.4, No.3 (June 2014)**

***Abstract***

*Conventional reliability theory is based mainly on probist reliability, which uses a binary state assumption and classical probabilistic distributions, which is often inappropriate for handling the uncertainty and imprecision of reasoning processes in real world applications due to lack of sufficient probabilistic information. To overcome this situation, Fuzzy rule based methods have been considered useful in the evaluation of reliability of the system as these methods allow the modeling and computation of the heuristic knowledge and linguistic information. In this paper, a four unit gracefully degradable system is developed and analyzed with respect to the effects of repair and coverage factors on its reliability. On the other hand Markov Model has proven its applicability in handling degradation, multiple-failures, imperfect fault coverage and other sequence dependent events, the reliability of the aforesaid system has been computed by incorporating the features of both Markov and Fuzzy Linguistic Modeling approaches.*

***Keywords:*** *Probist Reliability; Coverage Factor; Degradable System; Fuzzy Linguistic Modeling; Markov Model.*

## 1. Introduction

A gracefully degradable computing system is one which can demonstrate performance degradation (K.Y. Cai *et al*, 1991). The system can bear multiple faults in processing elements and/or links and still function with an acceptable performance degradation. The assumption that the system has only two performance levels, fully functioning or failed, holds for systems that have no redundancy. Such systems may be considered working to a certain degree at different states of its performance degradation during its transition from fully working state to completely failed state. The degree may be any real number between 0 and 1, where 0 indicates failed state and 1 indicates working state of the system. Thus, each state provides some performance level (which would be zero for complete failure). Therefore, a degradable system can have several reduced operational states between being fully operational and having completely failed (G. Haring *et al*, 2000).

In traditional reliability estimation the failure rate is modeled by probability theory and it requires large amount of data for estimating the failure rate. But in real world, the failure data sometimes cannot be recorded or collected precisely due to human errors or some unexpected situations. Consequently, some uncertainties are associated with component indices due to lack of up gradation of data (H. C. Wu, 2004). Due to this inaccuracy and uncertainty of data, the estimation of precise values of probability hence reliability becomes very difficult in many systems. Also, all the real world's systems possess uncertainties to a large extent and there are other unknown phenomena that cannot be modeled mathematically at all (M. R. Emami *et al*, 1998). The basic issue to address here is- which technique to use both for largeness tolerance and for largeness avoidance. This opens up the possibility of new techniques for reliability using Markov or Stochastic modeling and fuzzy logic techniques.

Fuzzy Logic is based on the concept of fuzzy set formulated in fuzzy set theory by Lotfi Zadeh, in 1965. Fuzzy set theory provides a mathematical framework for the systematic treatment of vagueness and imprecision. After the inception of the notion of fuzzy sets, several investigators paid attention to applying the fuzzy sets theory to reliability analysis. The collection of papers gave many different approaches to fuzzy reliability (T. Onisawa *et al*, 1995). Viertl (1989) considered the estimation of the reliability function when observed time data was fuzzy.

Markov modeling has long been accepted as a fundamental and powerful technique for reliability analysis which helps system designers surmount the mathematical computations that have previously prevented effective reliability analysis (J. Pukite *et al*, 1998). The basic concepts of Markov processes were applied in power system reliability, particularly to the 2-state fluctuating environment condition for simple configurations (R. Billintion *et al*, 1968). Fuzzy Markov models are proposed to incorporate the uncertainties associated with transition rates or probabilities. A fuzzy Markov model based method was described for determination of fuzzy state probabilities of generating units including the effect of maintenance scheduling (D. K. Mohanta *et al* 2005). A real-time power system reliability analysis method was

*Corresponding author: **Kalika Patrai**

developed based on a fuzzy Markov model (M. Tanrioven *et al*, 2006). Fuzzy logic is used in the Markov model to describe both transition rates and temperature-based seasonal variations, which identifies multiple weather conditions. In order to develop an Inference system based on fuzzy approach introduced the concept of fuzzy sets was introduced to model linguistic-like variables (G. Maglara *et al*, 1997). It has been found a wide range of applications in dealing with uncertainties involving vagueness, subjectivity, incompleteness, and imprecision in nature. Fuzzy set based methods have been proved to be effective in handling multiple types of uncertainties in different areas, including reliability engineering (A. Kanagawa *et al*, 1990; J. Wang *et al*, 1998; M. S. Moustafa, 1997). The main objective of this article is to develop a rule based fuzzy system, combining coverage factor with failure and repair rates, to estimate the overall reliability of degradable computing system.

## 2. System Modeling and Reliability Analysis

In the last two decades, the problem of reliability was intensively studied. Many research efforts were conducted to study various reliability models & evaluation methods, where the system failures are determined only by their component failures due to malfunctions, or their degradations beyond the maximum acceptable limit (B. S. Dhillon *et al*, 1997; C. J. Conrad *et al*, 1987; J. Xue *et al*, 1997). In the face of traditional modeling distinctions, we consider an important class of computing systems wherein the system performance is "degradable" that is depending on the history of the system's structure, internal state, and environment during some specified "utilization period" T, the system can exhibit one of several worthwhile levels of performance (J. F. Meyer, 1980).

Gracefully degradable systems may use all failure-free modules to execute tasks. Modeling needs for (gracefully) degradable systems were first investigated (B. R. Borgerson *et al*, 1975) in connection with their analysis of PRIME system (H. B. Baskin *et al*, 1972). As the number of system components and their failure modes increase, there is an exponential increase in system states, making the resulting reliability model more difficult to analyze. The large number of system states makes it difficult to interpret state probabilities, and to conduct sensitivity analyses. .

Markov models are frequently used in reliability & maintainability analysis where events, such as the failure or repair of a module, can occur at any point in time. A Markov model breaks the system configuration into a number of states and develops the probability of an item being in a given state, as a function of the sequence through which the item has traveled. The Markov process can thus easily describe degraded states of operation, where the item has either partially failed or is in a degraded state where some functions are performed while others are not (http://rac.alionscience.com). Also, when the coverage factor (the capacity of failure occurrence detection in a module of a system) is considered, the Markov model becomes a more suitable modeling approach, since the covered and uncovered failures of components are supposed to be mutually exclusive events. The present study is based on the Markov model of a gracefully degradable computing system, where the reliability of the system is computed using both the Markov and fuzzy modeling approaches and the results have been compared with respect to the effects of failure, repair and coverage factors.

### *3.1 Model Description*

To begin with the model, we consider a redundant degradable computing system with four identical and independent modules. Each module has only two states: failed or functioning. The time to breakdown for each module follows an exponential distribution with parameter λ. The system has a coverage factor c. When failure comes to same module, the system immediately takes reconfiguration operation with negligible time, to remove logically the faulty module whereas other fault free modules continue to do their work if the reconfiguration operation is performed successfully.

### *3.2 Design and Characteristics of the Model*

Let $S_i$ represents the system state that four active (operational) modules are available. The system may then have five states: $S_0$, $S_1$, $S_2$, $S_3$, and $S_4$. The system with five states is shown in Figure-1. The system in Figure-1 represents the Markovian transition among the system states, where c represents the system coverage factor, the success probability of a reconfiguration operation.

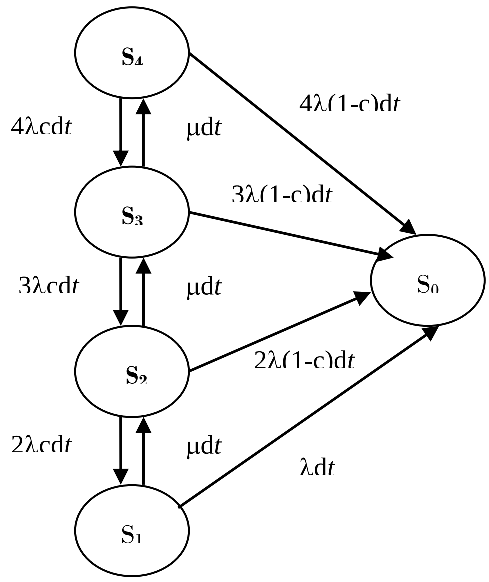


**Fig. 1** Markov model for a gracefully degradable computing system

The differential equations describing the system are

$$\frac{dP_4}{dt} = -4\lambda P_4 + \mu P_3$$

$$\frac{dP_3}{dt} = -3\lambda P_3 + 4c\lambda P_4 + \mu P_2$$

$$\frac{dP_2}{dt} = -2\lambda P_2 + 3c\lambda P_3 + \mu P_1$$
$$\frac{dP_1}{dt} = -\lambda P_1 + 2\lambda c P_2$$
$$\frac{dP_0}{dt} = \lambda P_1 + 2\lambda(1-c)P_2 + 3\lambda(1-c)P_3 + 4\lambda(1-c)P_4$$

The system is initially in state 4, thu $P_4(0)=1, P_3(0)=0, P_2(0)=0, P_1(0)=0, P_0(0)=0$. To obtain the reliability behavior of repairable system, the Laplace transform equations of $P_n(t)$, n = 0, 1, 2, 3 and 4 can be obtained using Figure-1 with Appendix A, where $P_n(t)$ is the probability of the system being in $n^{th}$ state at time t.

Assume that the state $S_0$ is systems down state. Let Z be the random variable representing the time to failure of the system, then $P_0(t)$ is the probability that the system fails at, or before, time t. Thus, the reliability function can be expressed as

$$R_Z(t) = 1 - P_0(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t), \qquad t \geq 0.$$

The conventional reliability of the system for different combinations of failure-rate, repair-rate and coverage factor over time period t are shown in Table-1 and plotted in Fig.-2(a)-2(b).

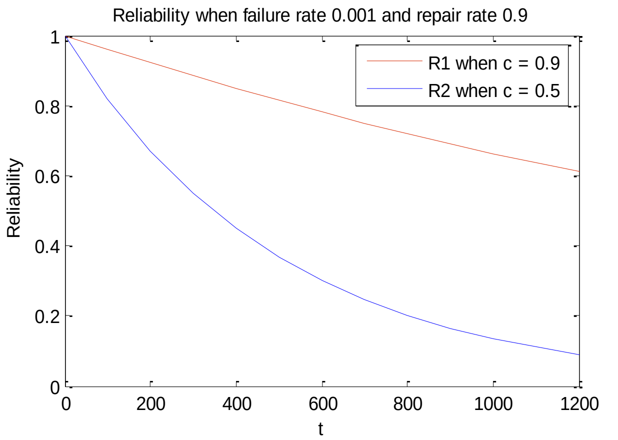


**Fig. 2(a)** Reliability index verses time period for λ = 0.001 and μ = 0. 9

## 3. Fuzzy Inference System

Fuzzy inference system (FIS) ( L. X. Wang *et al*, 1992) is a method based on the fuzzy theory which maps the input values to the output values and is an effective tool to address uncertainty due to imprecision and vagueness, especially when the data availability is limited. The mapping mechanism is based on some set of rules, a list of IF-THEN statements.

There are five steps in a fuzzy inference system. These steps are (i) fuzzification of the input variables where crisp set of input data are gathered and converted to a fuzzy set using fuzzy linguistic variables, fuzzy linguistic terms and membership functions, (ii) application of the fuzzy operator (AND or OR), if any, in the antecedent, (iii) implication from the antecedent to the consequent (formation of IF-Then rules), (iv) aggregation of the consequent across the rules and (v) defuzzification, to obtain a crisp value from fuzzy.

This fuzzy approach of the Markov model allows uncertainty-based parameters in the mathematical reliability evaluation. These parameters incorporate all the uncertainty about their values, and are modeled as fuzzy numbers. For the given system let $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{c}$ be the fuzzy parameters for failure rate, repair rate and coverage factor respectively. Every time the reliability index of the system will be governed by these three parameters.

**Table 1** System reliability behavior for different repair rates & coverage factors

| Operating Time t (in hrs.) | λ =.001 μ =.9, c =.9 R(t) | λ =.001 μ =.9, c =.5 R(t) | λ =.001 μ =.5, c =.5 R(t) | λ =.001 μ =.5, c =.9 R(t) |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 100 | 0.9598 | 0.8186 | 0.8184 | 0.9591 |
| 200 | 0.9212 | 0.6700 | 0.6698 | 0.9198 |
| 300 | 0.8842 | 0.5485 | 0.5482 | 0.8821 |
| 400 | 0.8487 | 0.4489 | 0.4486 | 0.8459 |
| 500 | 0.8145 | 0.3675 | 0.3672 | 0.8113 |
| 600 | 0.7818 | 0.3008 | 0.3005 | 0.7780 |
| 700 | 0.7504 | 0.2462 | 0.2459 | 0.7461 |
| 800 | 0.7202 | 0.2015 | 0.2013 | 0.7155 |
| 900 | 0.6912 | 0.1650 | 0.1647 | 0.6862 |
| 1000 | 0.6634 | 0.1350 | 0.1348 | 0.6581 |
| 1100 | 0.6368 | 0.1105 | 0.1103 | 0.6311 |
| 1200 | 0.6112 | 0.0905 | 0.0903 | 0.6052 |

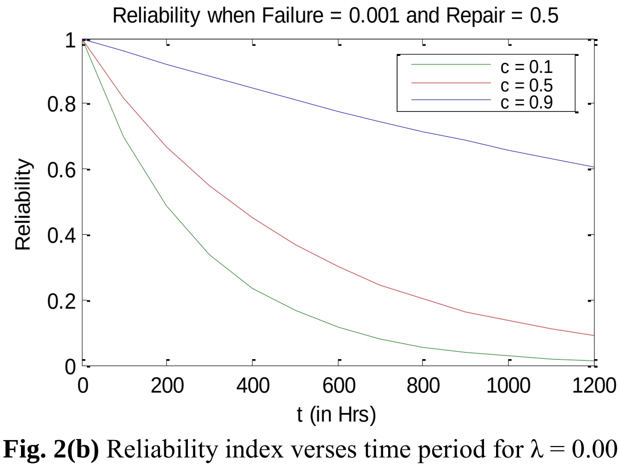


**Fig. 2(b)** Reliability index verses time period for λ = 0.001 and μ = 0. 5

*4.1 Fuzzy Rule Based Modeling*

Let $A_1$, $A_2$ and $A_3$ represent the fuzzy sets for $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{c}$. At a specific instant of time, all the measured data for $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{c}$ are fuzzified by membership function to their associate fuzzy sets $A_1$, $A_2$ and $A_3$.
We define $A_1$, $A_2$ and $A_3$ into linguistic spaces as follows:
$A_1$ = {Low, Medium, High}
$A_2$ = {Slow, Medium, Fast}
$A_3$ = {Imperfect, Moderate, Perfect}

Reliability Index = {Poor, Standard, High}

The fuzzy rules are in the format:
If (antecedent, related to the failure rate, coverage factor and repair rate) then (consequent, the reliability index).
Some of the rules which are formed for the system are given below:

1. If (Failure-rate is Low) and (Coverage-factor is Imperfect) and (Repair-rate is Slow) then (Reliability is Poor).
2. If (Failure-rate is Low) and (Coverage-factor is Moderate) and (Repair-rate is Medium) then (Reliability is High)
3. If (Failure-rate is medium) and (Coverage-factor is Moderate) and (Repair rate is medium) then (Reliability is Standard).
4. If (Failure-rate is Low) and (Coverage-factor is Perfect) and (Repair-rate is Fast) then (Reliability is High).

### *4.2 The Process of Aggregation*

A decision is based on the testing of all the rules in a fuzzy inference system. A combination of the rules is necessary for decision-making. Aggregation is the process by which the fuzzy sets that represent the outputs of each output are combined into a single fuzzy set. The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule. By choosing the appropriate rules from the rule-base, various graphs have been plotted.

### *4.3 Construction of Graphs*

The model for estimating the reliability of the system designed using fuzzy IF-THEN rules has been implemented on Fuzzy Logic Toolbox of MATLAB® [25]. The membership function graphs for failure-rate and reliability index have been shown in Figures 3(a) and 3(b).The graphs obtained for system reliability with respect to failure rate under moderate and perfect coverage are given in Figures 4(a) and 4(b).

## 5. Results and Discussion

Graphs for both, conventional reliability and fuzzy reliability have been plotted and discussed. From Figure 4(a), it is evident that if the coverage factor is 'moderate' [around 0.5] and repair rate is 'slow' [around 0.2] then reliability index is 'standard' at 'low' failure rate while it is ‘poor’ when failure rate is ‘high’. However, it may be seen that the reliability index is 'high' at 'medium' and 'fast' repair rates for 'moderate' coverage factor and ' low' failure rate. This trend remains same for 'fast' repair rate but reliability index becomes 'standard' for 'medium' repair rate. Whereas from Fig.-2(a) the conventional reliability calculated for ‘high’ failure-rate is ‘poor’ with ‘high’ repair rate and ‘moderate’ coverage factor and it can be seen from Fig.-2(b) that the reliability index for ‘moderate’ coverage and ‘medium’ repair is ‘high’ for ‘low’ failures and becomes ‘poor’ for ‘medium’ and ‘high’ failure rates.

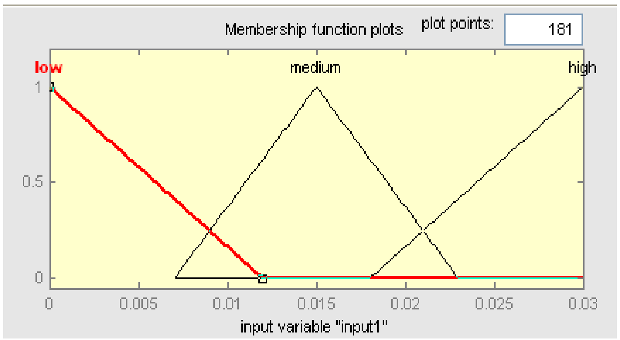


**Fig. 3(a)** Membership function for failure-rate

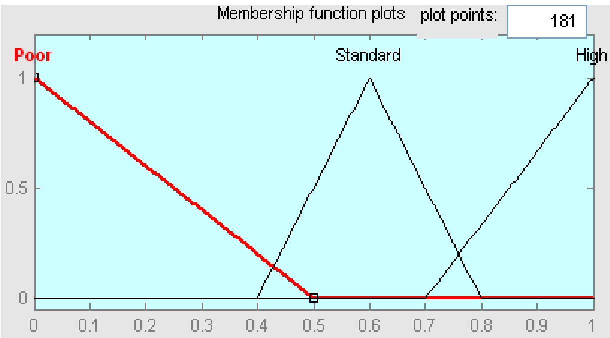


**Fig. 3(b)** Membership function for reliability-index

If the coverage is 'perfect' as shown in Fig. 4(b), the reliability index is 'standard to high' at 'slow' repair rate for 'low' failure rate while it is 'standard' at 'medium' and 'high' failure rates. For ‘fast’ repair rate the reliability index is ‘high’ for all types of failure rates whereas in conventional model reliability for ‘fast’ repair rate (around 0.9) is ‘high’ at ‘low to medium’ failure rates and ‘standard’ at ‘high’ failure rates. Thus the comparison between the Markov and fuzzy rule based models, in predicting the reliability index for different coverage factors, reveals fairly good agreement.

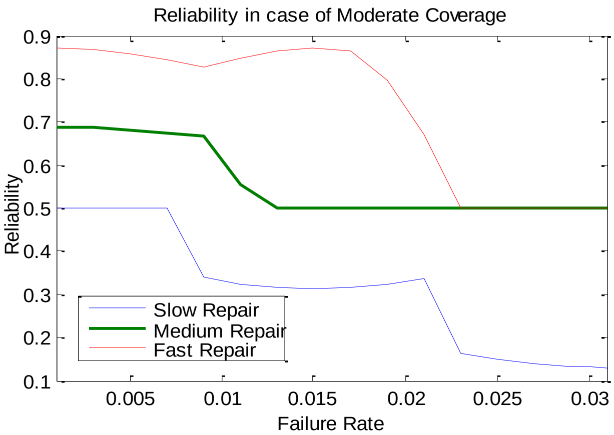


**Fig. 4(a)** Reliability estimation in case of ‘Moderate’ coverage factor

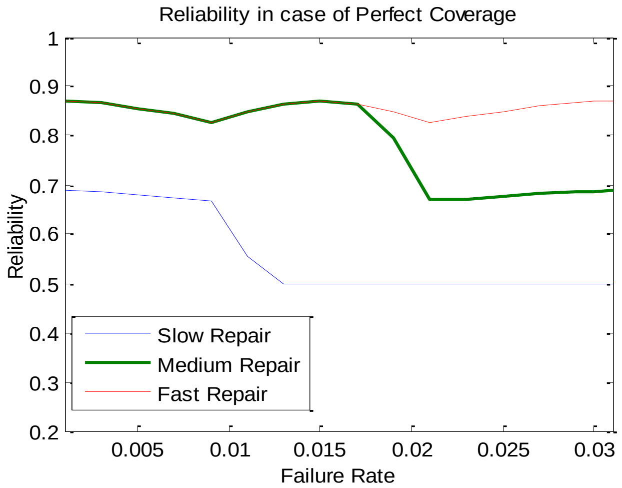


**Fig. 4(b)** Reliability estimation in case of 'Perfect' coverage factor

## 5. Conclusion

A fuzzy rule based modeling approach is presented for modeling and dealing with uncertainties in gracefully degradable computing systems. The main objective of the present work was to develop a Fuzzy Inference System for prediction of reliability of the system as a function of failure rate, coverage factor, and repair rate. The model results show that in case of 'low' failure rate the reliability of the system increases sharply with the 'perfect' coverage and 'fast' repair whereas in case of 'high' failure rate, the reliability of the system remains 'standard' with 'moderate' coverage factor and 'medium' repair rate.

Even though the results show fairly good agreement with the conventional one, the proposed model can be further examined for medium and higher failure rates to see its relevance with the traditional one.

## Appendix A

The Laplace Transform Equations for a Gracefully Degradable System

Assume that the process is initiall in state $S_4$, so that $P_4(0) = 1$, $P_3(0) = 0$, $P_2(0) = 0$, $P_1(0) = 0$ and $P_0(0) = 0$. Refereeing to the state transition diagram as shown in Fig. 1, the system differential equations using Laplace transform are obtained and given by

$$s\tilde{P}_4(s) - 1 = -4\lambda\tilde{P}_4(s) + \mu\tilde{P}_3(s)$$
$$s\tilde{P}_3(s) = -3\lambda\tilde{P}_3(s) + 4c\lambda\tilde{P}_4(s) + \mu\tilde{P}_2(s)$$
$$s\tilde{P}_2(s) = -2\lambda\tilde{P}_2(s) + 3c\lambda\tilde{P}_3(s) + \mu\tilde{P}_1(s)$$
$$s\tilde{P}_1(s) = -\lambda\tilde{P}_1(s) + 2\lambda c\tilde{P}_2(s)$$
$$s\tilde{P}_0(s) = \lambda\tilde{P}_1(s) + 2\lambda(1-c)\tilde{P}_2(s) + 3\lambda(1-c)\tilde{P}_3(s) + 4\lambda(1-c)\tilde{P}_4(s)$$

This system of linear equations can be solved to yield

$$\tilde{P}_4(s) = \frac{s + 3\lambda + \mu}{(s+4\lambda)(s+3\lambda+\mu) - 4\lambda\mu c}$$

$$\tilde{P}_3(s) = \frac{4\lambda c}{(s+4\lambda)(s+3\lambda+\mu) - 4\lambda\mu c}$$

$$\tilde{P}_2(s) = \frac{12\lambda^2c^2(s+\lambda+\mu)}{\{(s+4\lambda)(s+3\lambda+\mu) - 4\lambda\mu\, c\}\{(s+\lambda+\mu)(s+2\lambda+\mu) - 2\lambda\mu\, c\}}$$

$$\tilde{P}_1(s) = \frac{24\lambda^3c^3}{\{(s+4\lambda)(s+3\lambda+\mu) - 4\lambda\mu\, c\}\{(s+\lambda+\mu)(s+2\lambda+\mu) - 2\lambda\mu\, c\}}$$

## References

K. Y. Cai, C. Y. Wen and M. L. Zang (1991), Fuzzy reliability modeling of gracefully degradable computing systems, *Reliability Engineering and System Safety*, 33(1), 141-157.

G. Haring, C. Lindemann and M. Reiser (2000), Performance Evaluation: Origins and Directions, *Lecture Notes in Computer Science*, SpringerBerlin.

H. C. Wu (2004), Fuzzy reliability estimation using Bayesian approach, *Computers & Industrial Engineering* 46, 467–493.

M. R. Emami, I. B. T-3, Urksen, and A. A. Goldenberg (1998), Development of a Systematic Methodology of Fuzzy Logic Modelling, *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 3, pp. 346-361.

L. A. Zadeh (1965), Fuzzy set, *Information Control*, 8, 338-353.

T. Onisawa and J. Kacprzyk (1995), Reliability and Safety Analyses Under Fuzziness, *Heidelberg: Physica-Verlag*.

R. Viertl (1989), Estimation of the reliability function using fuzzy life time data, *In P.K. Bose et al., editor, Quality for Progress and Development*, Wiley Eastern, New Delhi.

J. Pukite, P. Pukite (1998), Modeling for Reliability Analysis: Markov Modeling for Reliability, Maintainability, Safety, and Supportability Analyses of Complex Systems, *IEEE Press Series on Engineering of Complex Computer Systems*.

R. Billinton, K. E. Bollinger (1968), Transmission System Reliability Evaluation Using Markov Processes, *IEEE Transactions on Power Apparatus and Systems*, Vol. Pas-87, No. 2.

D. K. Mohanta, P. K. Sadhu, R. Chakrabarti (2005), Fuzzy Markov model for determination of fuzzy state probabilities of generating units including the effect of maintenance scheduling, *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 2117–2124.

M. Tanrioven, Q.H. Wu, D.R. Turner, C. Kocatepe, J. Wang (2006), A new approach to real-time reliability analysis of transmission system using fuzzy Markov model, *Electrical Power and Energy Systems*, 26, 821–832.

G. Maglara, E. Nikolaidis, R. T. Haftka, H. Cudney, (1997), Analytical-experimental comparison of probabilistic methods and fuzzy set based methods for designing under uncertainty, *Structural and Multidisciplinary Optimization*, 13(2-3), 69-80.

A. Kanagawa, H. Ohta (1990), Fuzzy design for fixed-number life test, *IEEE Transaction on Reliability*, 39(3), 76-83, 1990.

J. H. Wang, J. Liittschwager, T. Raz (1998), Censored life-date analysis using linguistic variables, *IEEE Transactions on Reliability*, 37(1), 111-116.

J. H. Wang, J. Liittschwager, T. Raz (1999), Equipment life estimation using fuzzy set theory, *In: G. W. Evans, ed: Fuzzy Methodologies for Industrial and System Engineering*, 197-211.

M. S. Moustafa(1997), Reliability analysis of k-out-of-n: G systems with dependent failures and imperfect coverage, *Reliability Engineering and System Safety*, vol. 58, pp. 15–17.

B. S. Dhillon, N. Yang (1997), Comparisons of block diagram and Markov method system reliability and mean time to failure results for constant and nonconstant unit failure rates, *Microelectron. Reliability*., vol. 37, pp. 505–509.

C. J. Conrad, N. H. McClamroch (1987), The drilling problem: a stochastic modeling and control example in manufacturing, *IEEE Transactions on Automatic Control*, vol. AC. 32, pp. 947- 958.

J. Xue, K. Yang (1997), Upper & lower bounds of stress-strength interference reliability with random strength-degradation, *IEEE Transactions on Reliability*, vol. 46, pp. 142–145.

John F. Meyer (1980), On evaluating the performability of degradable computing system, *IEEE Transactions on Computers*, vol. C-29,No. 8, pp 720-731.

B.R. Borgerson, R.F. Freitas (1975), A reliability model for gracefully degrading and standby-sparing systems, *IEEE Transactions on Computers*, vol. C-24, pp517-525.

H.B. Baskin, B.R. Borgerson, R. Roberts (1972), PRIME-A modular architecture for terminal-oriented systems, Spring Joint Computer Conference, *AFIPS Conference Proceedings*, vol. 40. Washington, DC: Spartan, pp. 431-437.

The applicability of Markov analysis methods to reliability, maintainability and safety, *Reliability Analysis Centre*, Mill Street, Rome, 2003. Available: http://rac.alionscience.com.

L. X. Wang, J. M. Mendel (1992), Generating fuzzy rules by learning from examples, *IEEE Transactions on Systems, Man and Cybernetics*, 22(6), 1414–1427.

Fuzzy Logic Toolbox for use with MATLAB®, U. S. A.: The MathWorks, Inc., 2007.