International Journal of Current Engineering and Technology
E-ISSN 2277 – 4106, P-ISSN 2347 - 5161

Available at http://inpressco.com/category/ijcet

Research Article

Influence of Soil Stiffness and of End Support Conditions of a Pile in its Buckling Force

Vlora Shatri[Ȧ*], Luljeta Bozo[Ḃ], Burbuqe Shatri[Ȧ] and Bajram Shefkiu[Ȧ]

[Ȧ]Department of Materials and Construction, Faculty of Civil Engineering & Architecture, University of Prishtina, Kosovo
[Ḃ]Department of Urban Planning and Environment Management, University of Polis, Tirana, Albania

Accepted 15 January 2014, Available online 01 February 2014, **Vol.4, No.1 (February 2014)**

Abstract

The aim of this work is to analyze the effects of soil stiffness and of pile end conditions into the buckling force of the pile fully embedded on ground and subject to only axial compression force. The pile buckling force is analyzed for the case when the modulus of soil reaction is k=0, k=const and $k=n_h \cdot x$. The software SAP 2000 is used for analysis.

Keywords: *Pile, buckling force, buckling length, modulus of soil reaction.*

1. Introduction

The phenomena of buckling is described as an instability of a completely straight column subject to an axial load when exceeding a certain value. Initially buckling was limited in defining/assessing the limiting value of the force beyond which a column becomes instable.

Resulting out of the nowadasy researches it is found that buckling of piles usually occures when when the same are embeded in a very soft soils such as the soft clays, very incoherent sands, etc. Mathematically and experimentally is established that vulnerability to the buckling of piles considerably decreases out of lateral limitations caoused by the surrounding soil and due to this the buckling phenomena is in most of soil types neglected.

2. Analysis of pile buckling and dtermination of critical buckling force

The pile buckling problem is closely related to the problem of a beam on an elastic foundation; therefore with the aim of calculating the response of a vertical pile fully embedded on ground and subject to an external load, the pile will be treated as a beam of an elastic foundation (Matlock, H. and L. C. Reese)

The critical buckling force of a pile in an environment that offers resistance to the lateral displacement depends on:
- pile length, *L*
- soil stifness, *K* and *Es*
- Conditions of the end restraint of the pile with the pad and the soil.

Flexural pile stiffness, EI.

Most of analithycal methods used to analyze buckling of piles are based on the theory of soil reaction modulus. The therm soil reaction modulus represents pressure P per unit area of the contact surface between the loaded beam or slab and the soil it is supported by and on which the load is transmitted on. The coeficient of the soil reaction, k, is the ratio between lateral soil pressure P, on each determined node of the contact surface and the deflection, y, coused by the respective force applied at that node:

$$k = \frac{p}{y} \quad \left[kN / m^2 \right] \tag{1}$$

2.1 Determination of pile buckling force for the case when k=0

A pile of the following end conditions will be considered: pinned on the head and pinned on the tip (*p-p*). The equation of flexure of a pile subject to axial load is:

$$E \cdot I \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + k \cdot y = 0 \tag{2}$$

Substituting *k=0* in equation (2) the following is obtained:

$$E \cdot I \cdot \frac{d^4 \cdot y}{d \cdot x^4} + P \cdot \frac{d^2 \cdot y}{d \cdot x^2} = 0 \tag{3}$$

Substituting $\alpha^2 = P/(EI)$, the differential equation (3) takes the following shape:

$$\frac{d^4 \cdot y}{d \cdot x^4} + a^2 \cdot \frac{d^2 \cdot y}{d \cdot x^2} = 0 \tag{4}$$

After a fourfold integration a differential equation of the following type will be obtained:

$$y(x) = C_1 \cdot \sin\alpha \cdot x + C_2 \cdot \cos\alpha \cdot x \tag{5}$$

*Corresponding author: **Vlora Shatri**

The constants of integration are determined out of the end conditions of a pile.

$$y(x=0)=0 \qquad C_2=0 \tag{6}$$
$$y(x=l)=0 \qquad C_1\cdot\sin\alpha\cdot l=0 \tag{7}$$

The equation (7) will be satisfied if $C_1=0$, and that is a trivial solution. A pile will not deflect and that means that $y(x)=0$ for entire range of x. This shows the position of the pile prior to buckling. Besides this the condition $C_1\cdot\sin\alpha\cdot l=0$ will be fulfilled for:

$$\sin\alpha\cdot l=0 \tag{8}$$

This is the case when:

$$\alpha=\frac{n\cdot\pi}{l},\quad n=1,2,3,.. \tag{9}$$

Using the substitution $\alpha^2=P/(EI)$ we may write:

$$P=n^2\cdot\frac{\pi^2}{l^2}\cdot E\cdot I \tag{10}$$

The smaller value of P for $n=1$ represents the critical buckling force of the *(p-p)* $P_{krit.}$.

$$P_{krit}=\frac{\pi^2}{l^2}\cdot E\cdot I \tag{10}$$

In Figure 1, are given four basic cases of piles with equal length and different end conditions as well as the curves that represent the flexural shape of the piles in case when k=0, while the expressions for calculation of buckling forces are given in Table 1. For the all four cases of buckling, the buckling length is taken:

$$l_0=\beta\cdot l \tag{11}$$

While the buckling force of the pile expressed through the buckling length is given:

$$P_{krit}=\frac{\pi^2}{l_0^2}\cdot E\cdot I \tag{12}$$

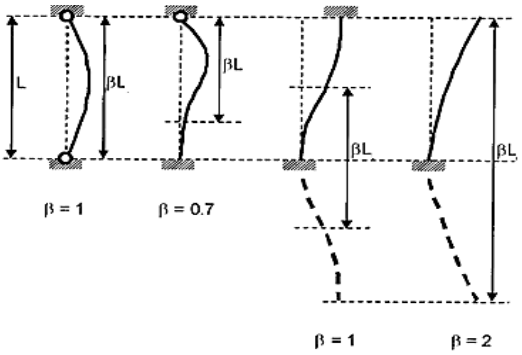

Figure 1 Buckling forces and buckling lengths of piles in a function of restraint conditions for the case when $k=0$

Table 1

$P_{krit}=\frac{\pi^2}{l^2}\cdot E\cdot I$	Pile pinned on the head – pinned on the tip
$P_{krit}=\frac{\pi^2}{(0.7\cdot l)^2}\cdot E\cdot I$	Pile pinned on the head – fixed on the tip
$P_{krit}=\frac{\pi^2}{(2.0\cdot l)^2}\cdot E\cdot I$	Pile free on the head – fixed on the tip
$P_{krit}=\frac{\pi^2}{(0.5\cdot l)^2}\cdot E\cdot I$	Pile fixed on the head – fixed on the tip

2.2 Determination of the pile buckling force when k=const

We will write again the buckling differential equation for the pile subject to axial compression force only:

$$E\cdot I\frac{d^4y}{dx^4}+P\frac{d^2y}{dx^2}+k\cdot y=0 \tag{2}$$

End conditions will be:

$$\begin{aligned} y(x=0)&=0;\quad y''(x=0)=0\\ y(x=l)&=0;\quad y''(x=l)=0 \end{aligned} \tag{13}$$

The solution of differential equation (13) will be:

$$y(x)=C\cdot\sin\left(\frac{n\cdot\pi\cdot x}{l}\right) \tag{14}$$

Solution of differential equation (2) for the force (P) will be:

$$P_{krit}=n^2\frac{\pi^2}{l^2}\cdot E\cdot I+\frac{1}{n^2}\cdot\frac{k\cdot l^2}{\pi^2} \tag{15}$$

where:

$$P_E=\frac{\pi^2}{l^2}\cdot E\cdot I \tag{16}$$

The first part of the equation (15) corresponds to Euler's equation for buckling of a column while the second part reflects contribution of lateral restraint caused by the soil. In equation (15) it is taken that the relation force – deflection is linear and that the modulus of horizontal reaction of soil is constant within the depth, a fact that for most of soils does not prevail.

For a pile embedded in an elastic medium, a general expression for critical buckling force is:

$$P_{krit}=P_E\left(n^2+\frac{B}{n^2}\right) \tag{17}$$

where n is the number of semi-waves of a sinusoidal curve in the deflected shape of the column and

$$B=\frac{k\cdot l^4}{\pi^4\cdot E\cdot I} \tag{18}$$

It is easy to show that minimal value of P_{crit} corresponds to the value

$$n = \left[k \cdot (E \cdot I)^3\right]^{1/4} l / \pi \cdot E \cdot I = B^{1/4} \tag{19}$$

For the pile with end conditions such as free on the head-fixed on the tip, the Euler's force is:

$$P_E = \frac{\pi^2}{4 \cdot l^2} \cdot E \cdot I \tag{20}$$

We substitute n and P_E in equation (17), simplify and will obtain:

$$P_{krit} = \frac{1}{2}(k \cdot E \cdot I)^{1/2} \tag{21}$$

The critical buckling length of the pile is obtained from the condition:

$$\frac{\partial P}{\partial\left(\frac{l}{n}\right)} = 0 \tag{22}$$

$$l_0 = \pi \cdot \sqrt[4]{\frac{E \cdot I}{k}} \ [m] \tag{23}$$

From where it is seen that the buckling length or as it's called, the critical length contains the two elements that are: the pile element expressed through the stiffness of a pile EI, and the element soil, expressed through the design soil coefficient, k. If the pile is to stiff comparing to the soil, then the buckling length takes relatively high values, that means that pile load will cause considerable deflection/displacement. In contrary, if the soil is o stiff comparing to the pile, then the buckling length of the pile would be relatively small. This way, it is possible that for the system pile-soil to (determine) the buckling length and from the solution of the given problem define the point in which the deflection of the pile is equal to zero, hence beyond (downwards) this point the deflection is so small so as it can be neglected. If the pile is smaller than the buckling length we call it a stiff pile, at the other side, a pile with the buckling length greater than the buckling length is called a slender pile. With other words, as greater the ratio between the pile stiffness and the soil stiffness the greater pile length is required (necessary) in order the pile to be considered flexible.

For calculation of critical buckling force of the pile pinned at both ends, it is possible that in some cases we use the in-equation (Davisson, M. T.)

$$P_{krit} \geq 2 \cdot \sqrt{k \cdot E \cdot I} \tag{24}$$

Particularly for greater values of l, critical buckling force iscalculated sufficiently accurate according to the followingexpression (Shnell, W., Czerwenka, G.):

$$P_{krit} \approx 2 \cdot \sqrt{k \cdot E \cdot I} \tag{25}$$

If we compare the value of critical buckling force of the pile, pinned at both ends, for *k=const*, with a column of the same supporting conditions but that is not subject to lateral load along its length *l*, *(k=0)*, usually the critical buckling force is much greater than the buckling force of the column.

3. Pile buckling analysis by Davisson's Method

3.1Determination of a pile buckling force for the case when k=const.

If the differential equation (2) is divided by $E \cdot I$ we will obtain:

$$\frac{d^4 y}{dx^4} + \frac{P}{E \cdot I}\frac{d^2 y}{dx^2} + \frac{k}{E \cdot I} \cdot y = 0 \tag{26}$$

Further solution of the differential equation (26) will be made in a way that all parameters within the equation are taken as being constant. The solution of the equation (26) will be obtained in a non-dimensional form if taken:

$$R = \sqrt[4]{\frac{E \cdot I}{k}}, \quad Z = \frac{x}{R}, \quad Z_{\max} = \frac{L}{R} \tag{27}$$

this way:

L support length of a pile
R relative stiffness factor
EI pile stiffness
Z non-dimensional coefficient of depth

Differential equation (26) will take this shape:

$$\frac{d^4 y}{dz^4} + \frac{P \cdot R^2}{E \cdot I}\frac{d^2 y}{dz^2} + y = 0 \tag{28}$$

Taken that

$$U_{krit} = \frac{P_{krit} \cdot R^2}{E \cdot I} \tag{29}$$

U_{crit}, is the axial force coefficient.
We substitute the axial force coefficient in equation (29) and we obtain:

$$\frac{d^4 y}{dz^4} + U \cdot \frac{d^2 y}{dz^2} + y = 0 \tag{30}$$

The critical values of the axial force, U_{crit}, are taken from the solution of the equation (30) according to U, bay taking into account the end conditions and the pile length Z_{max} (Fig. 2).
The L/R parameter is often used to distingussh between the stiff pile and flexible:
L/R≤2: stiff
2<L/R≤4: mediate
L/R>4: flexible

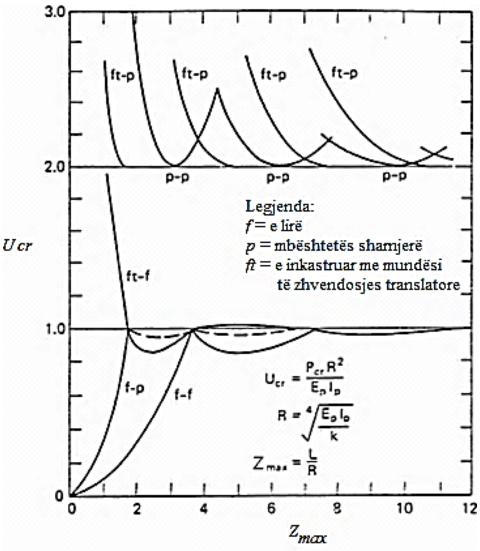

Figure 2 Buckling force as a function of pile length for *k=const* (Davisson, 1963)

3.2 Determination of buckling force of a pile for the case when $k=n_h{\cdot}x$.

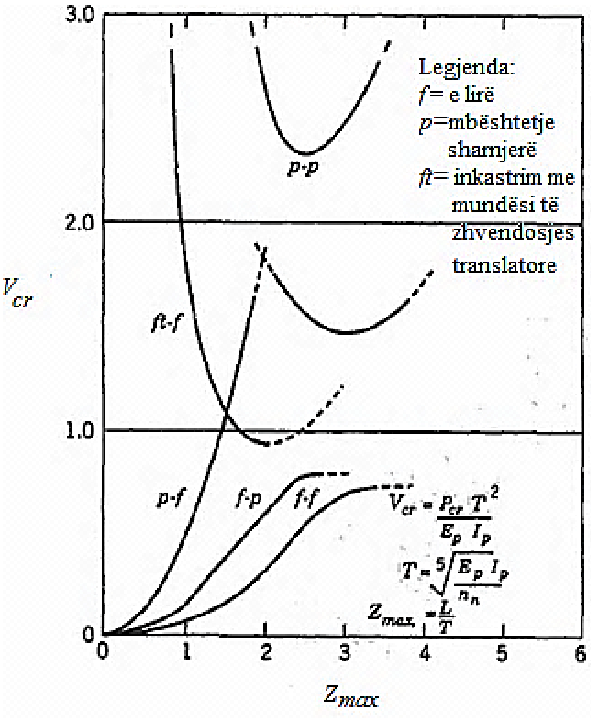

Figure 3 Buckling force in a funcion of the pile length for $k=n_h{\cdot}x$, (Davisson, 1963)

In the case when the soil reaction modulus is taken as $k=n_h{\cdot}x$, buckling of the pile will be influenced by end conditions and especially the end condition on the head of the pile since the pile tends to deflect where the lowest soil reaction modulus is. We substitute in equation (2) $k= n_h{\cdot}x$, then the equation will take this shape:

$$E\cdot I\frac{d^4y}{dx^4}+P\frac{d^2y}{dx^2}+n_h\cdot x\cdot y=0 \tag{31}$$

Substituting with:

$$T=\sqrt[5]{\frac{E\cdot I}{n_h}},\quad Z=\frac{x}{T},\quad Z_{\max}=\frac{L}{T} \tag{32}$$

Where:

T the relative stiffness factor
Z the depth non-dimensional coefficient
Z_{max} maximum value of coefficient of depth
Equation (31) will take the following shape:

$$\frac{d^4y}{dz^4}+\frac{P\cdot T^2}{E\cdot I}\frac{d^2y}{dz^2}+Z\cdot y=0 \tag{33}$$

If we denote with V, the coefficient of axial force $P{\cdot}T^2/EI$, later on:

$$V_{krit}=\frac{P_{krit}\cdot T^2}{E\cdot I} \tag{34}$$

By substituting the expression (34) in equation (33) we will have:

$$\frac{d^4y}{dz^4}+V_{krit}\frac{d^2y}{dz^2}+Z\cdot y=0 \tag{35}$$

Equation (35) is solved for V_{crit} as a function of Z_{max}, in a computational way, according to Davisson (1963). In Fig.3 is given the V_{crit}, in a function of Z_{max}, for a pile with these differing support conditions.

4. Calculation of buckling load with software SAP 2000

Linear analysis of buckling of piles with the use of SAP 2000 software requires instable shapes of structure due to the P-delta effect subject to a group of respective loads. The buckling analysis includes general solution of the problem of eigen values:

$$[K-\lambda G(r)]\Psi=0 \tag{36}$$

where it is:
K stiffness matrix,
G(r) required geometrical stiffness (P-delta) due to the load vector r,
λ diagonal matrix of eigen values, and
Ψ the matrix that corresponds to the eigen vector (modal shape)

Each couple of eigen value – eigen vector represents a shape of buckling of the structure. There are shapes for buckling identified in series from 1 to n that can be calculated by use of the software.

The eigen value λ is called the buckling factor. The load r, is to be multiplied with this factor so as to cause buckling in a given shape. This factor could be taken as a safety factor too: if the buckling factor is greater than one, the given load should be increasing to cause buckling, if this factor is smaller than one, the loud should be decreasing in order to prevent buckling. The buckling a factor could also be negative. This shows that the buckling will happen if the loads are opposite.

We can create various cases of linear buckling force. Combination of one or more modal loads and/or loads due to acceleration contained in a vector of loads (load vector) r, should be specified in any case. Using this software, the number of buckling shapes could be also specified in order to find a tolerance of the convergence. It is always recommended to seek more than one buckling shape since the first ten shapes may have very similar he buckling factors. A minimum of six buckling shapes are recommended out of what it is understood that they depend on the load.

With the aim of determining the pile buckling load according to software ***SAP 2000,*** the pile is divided into n equal pieces of length of Δx. The modulus of soil reaction is calculated for each node of the pile as in the Fig. 5.

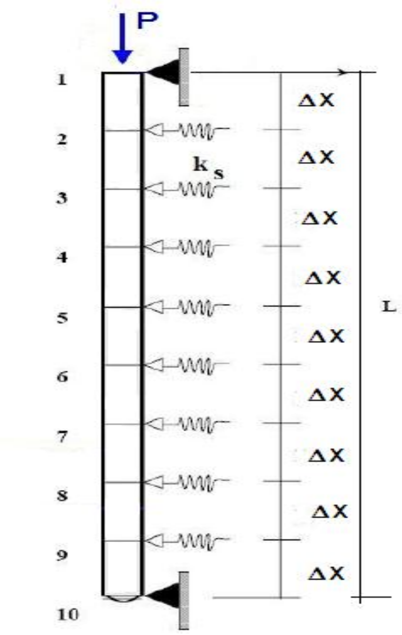

Figure 4 Division of pile on finite elelements length of Δx

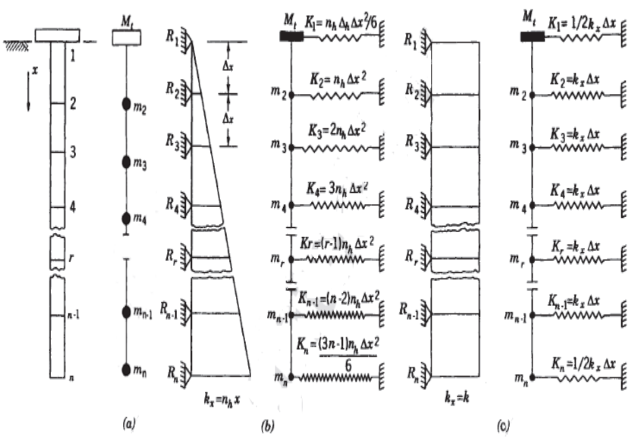

Figure 5 Discretization of the effect of interaction soil-pile. (a) Idealization of the structure of piles, (b) soil reaction modulus varying linearly and with the depth, (c) soil modulus varying proportionally with the depth (Chandrasekaran, 1974).

5. Results and Discussion

5.1. Influence of the soil stiffness in the buckling force in case when the soil stiffness increases linearly with the depth, $k=k_0+n_h{\cdot}x$ and with k_0=const. according to SAP 2000

In Fig. 6 are given the values of buckling forces calculated with the software SAP 200, where for the purpose of analysis is adopted a reinforced concrete pile of the following end conditions: pinned on the head – pinned on the tip, and of stiffness of $EI= 12120{,}9\ kNm^2$, C-25/30, *D=30cm*.

Diagrams of buckling force are given in fig.6 as a function of pile length L. In the first case the stiffness is adopted $k=100\ kN/m^2$ while in the other two cases the stiffness is taken $k=100\ kN/m^2$ and $n_h=100\ kN/m^3$ *as well as* $k=100\ kN/m^2$ and $n_h=500\ kN/\ m^3$. From Fig. 6 we notice that buckling shapes differ between with an increase of a pile length and we also have an increase of buckling force as the value n_h increases.

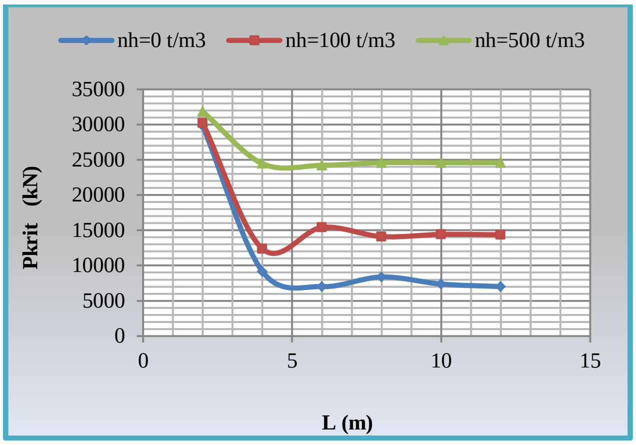

Figure 6 Values of critical buckling force for a pile with end conditions, pinned in the bottom and pinned on the head calculated with the software *SAP 2000* for $k=100\ kN/m^2$

5.2 Effects of increased values of k_0 in the buckling force when n_h=constant

In Fig.7 are given the diagrams of buckling forces $P_{crit,}$ as a function of pile buckling length L, stiffness of $1212{,}9\ knm^2$, $n_h=100kn/m^3$, and $k_0= 0,\ 100,\ 500$ and $1000\ t/m^2$. For these cases too, the buckling shapes vary with an increase of the pile length and the buckling forces increase with an increase of k_0 *while* n_h is kept constant. Comparing the given diagrams in Fig. 7 with those in Fig. 6 could be concluded that the values of buckling forces are bigger for cases when $k=100\ kN/m^2$ and $n_h=0,\ 100$ and $500\ kN/\ m^3$ comparing with the cases for $n_h=100\ kN/m^3$ and $k_0\ = 0,\ 100, 500,$ and $1000t/m^2$.

5.3 Influence of boundary conditions on the pile buckling force

In Fig. 8 are given the diagrams of critical buckling forces of a pile as a function of the pile length L for the case when $k=100t/m^2$ and $n_h=100t/m^3$ for two end conditions:

(p-p) – pinned on the head and pinned in the tip of the pile, and *(F-F)* – fixed in the head and fixed in the tip of the pile.

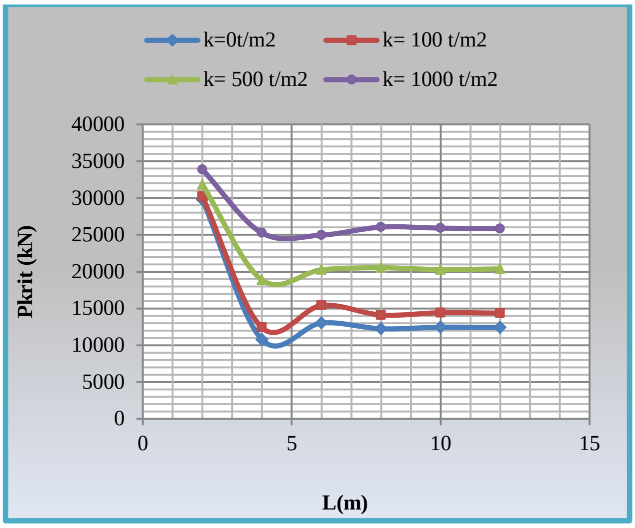

Figure 7 Values of critical buckling forces based on SAP 2000 software for the case when pile end support conditions pinned on the tip – pinned on the head when n_h=100kNm^3

From Fig.8 it is noticed that the buckling force relatively decreases with an increase of a pile length. For a pile with the end conditions (F-F), the values of buckling forces are considerably bigger comparing to those of the pile with the end conditions (p-p).

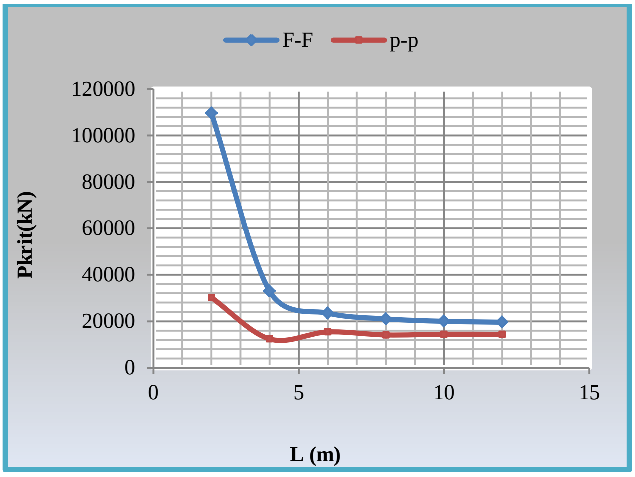

Figure 8 Values of critical buckling force for a pile with end different conditions, when $k=100t/m^2$ and $n_h=100t/m^3$ based on SAP 2000 software

Conclusions

The modal shapes of buckling of piles analysed with SAP 200 software are influenced by pile length since they differ between with an increase of the length of the pile.

The buckling force of a pile is influenced by the pile stifnnes. The buckling forces are increased when n_h values are increased while k remains constant . Buckling force values are also increased in cases when the n_h is kept constant and k increased. For a pile of the same length, the bigger buckling force is obtained when n_h remains constant and the k is increased as in the opposite case. Hence the soil stifness directly ifluences the buckling force of th epile.

The end support conditions are also influencing the pile buckling force. The buckling force is maximal for the pile with the end conditions fixed-fixed (F-F) and minimal for the end conditions pined-pined *(p-p)*.

If the pile is to stiff comparing with the soil, then the buckling length has relatively larger value, this means that the pile load will cause considerable deflections. In contrary if the soil is to stiff comparing to the pile, then the buckling length of the pile would be relatively shorter. Beside the above mentioned factors for the pile buckling force, the pile stifness itself has its influence too.

References

Hetenyi, M. I., (1960) Beam on Elastic Foundation, University of Michigan Press,

Matlock, H. and L. C. Reese, Generalized Solutions for Laterally Loaded Piles, ASCE Transactions, Paper No.3370

Georg Maybaum, Petra Mieth, Wolfgang Oltmanns, Rainer Vahland. (2009):Verfahrenstechnik und Baubetrieb im Grund- und Spezialtiefbau,

Davisson, M. T., (July 1963) Estimating Buckling Loads for Piles, Proceedings, 2nd Pan American Conference on Soil Mechanics and Foundation Engineering.

Davisson, M. T. and H.L.Gill, (1963) Laterally Loaded Piles in a Layered Soil System, ASCE, Proceedings of ASCE SM3.

Davisson, M. T. and K. E. Robinson, (1965) Bending and Buckling of Partially Embedded Piles, Proceedings of the 6th International ConFerence on Soil Mechanics and Foundation Engineering

Shamshar Prakash and Hari D.Sharma, (1990) Pile Foundations in Engineerin PracticeCopyright.

Norbert Vogt prof. Dr.-ing., Stefan Vogt dip.-ing., Christian Kellner dr.-ing. Aug 2009 Buckling of Slender Piles in Soft Soils

A. Siva Reddy, A.M.ASCE,(Assoc.Prof., Civ. and Hydr. Engrg. Dept..Indian Inst. of Sci., Bangalore, India) and Arun J. Valsangkar. (Sr. Res. Felloww, Civ. and Hydr. Engrg.Dept..Indian Inst. of Sci., Bangalore, India) December 1970 Buckling of Fully and Partially Embedded Piles

Sriram Kalaga, December. 2001Critical buckling loads of concrete piles in clay Journal of Structural Engineering No.2R-14 Vo1.28 No.3 October- pp. J 53-156

T.R.Aldridge, S.Bhattacharya, and T.M.Carrington,(Sept 2005): Buckling considerations in pile desing Proceedings of the International Symposium.on Frontiers in Offshore Geotechnics (IS-FOG 2005), 19-21Perth, WA, Australia.

Prakash, Sally, (1985) Buckling Loads for Fully Embedded Piles, M.S Thesis University of Missouri-Rolla

Prakash,Sally, (1987), Buckling Loads of Fully Embedded Piles, Int. J. Computer Geotech. Vol.4, pp.61-83

Shnell, W., Czerwenka, G.: Einführung in die Rechenmethoden des Leichtbaus. Bibliographisches Institut Mannhaeim, Band 2)