

Research Article

Mathematical Analysis for Small-Scale Osmotic Power Generation

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Abstract

According to the 'Guardian' the rise in the carcinogens and other harmful compounds are on rise since 1971. The increase have been noted to be 2% per year. United States contribute to 10% increase between 1999 and 2011. Harnessing energy from various renewable sources is the need of the hour and this paper puts forward a method which is in research stage at Statkraft*. The paper gives a mathematical investigation for it's use for small scale applications and the methodology is further generalized. The results obtained can be used for respective applications as they are largely pertinent.

Keywords: Osmotic Power, Mathematical Investigation, Buckingham pi theorem.

1. Introduction

When two solutions are separated by semipermeable membrane which allows only passage of solvent molecules, the solvent (generally water) flows from less concentration side to more concentration side. This phenomenon is called as osmosis. Osmosis is explained by the term osmotic pressure. It is given by the Vant Hoff formula:

$$\pi = k * c * t$$

This gradient energy is utilized in the method of Pressure Retarded Osmosis (PRO) as a renewable source of energy for power production.

Osmotic power as a source of energy started from the researches done by Prof. Sidney Loeb in 1973 at the Ben-Gurion University of the Negev, Beersheba, Israel. The use of this method has been increasing over the years (though on a research stage), because of the opening of the an osmotic power plant by the Statkraft, a Norwegian state-owned power company, in 2009.

The concept of PRO is used for generation of electric power similar to that of a hydro-electric power plant using a turbine coupled to a generator. The more concentration water and less concentration water separated by the semi permeable membrane results in increase in level of water on more concentration side. This head is utilized for generation of power. The flow of solvent through the membrane is due to the presence of difference in osmotic pressure which is a result if salinity gradient.

In the following paper this large power plant has been reduced for household applications with a theoretical proof using a mathematical method viz. Buckingham pi theorem.

2. Methodology

The method opted for the mathematical proofs is dimensional analysis by Buckingham Pi Theorem. It gives dimensionless numbers which are functions of parameters used for calculating certain relations between the factors.

3. Concept of Osmosis

Figure 1 represents the concept of osmosis. On the right hand side of the tube there is high concentration solution and on the left side there is less concentration solution. Due to the presence of difference in osmotic pressure the solvent flows from right hand side to left hand side and thus giving a rise in water level on right side. This process is called as osmosis.

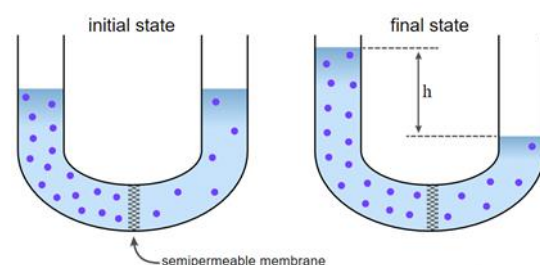


Fig 1

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4. Pressure Retarded Osmosis

When the flow of solvent due to osmosis is suppressed by an external pressure on the high concentration side, the process is called as pressure retarded osmosis (PRO). The osmotic flow is reduced as the external pressure opposes the conventional flow of solvent. Fig 2 shows the trend in water flux according to the external pressure. When external pressure ΔP is zero water flux is maximum and the process is termed as forward osmosis. When it is gradually increased the water flux declines progressively. For power generation ΔP is kept to certain value and increase in water head is used to generate power.

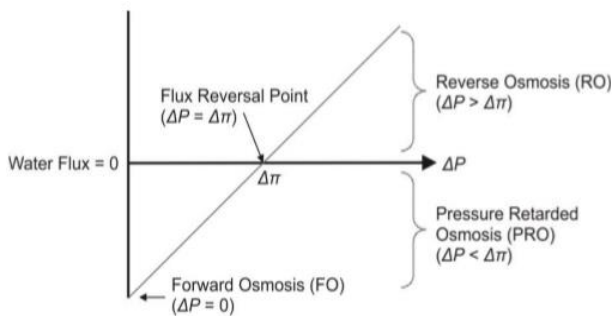


Fig 2

5. Brief about Statkraft osmotic power plant

Power generation using osmotic power or salinity gradient was first attempted by Statkraft at Tofte in Norway. It is operated by SINTEF Energy Research, a research division of SINTEF Group.

Mette-Marit the crown princess of Norway released the prototype on 24th November 2009. The prototype plant has a designed capacity to generate 10kW of electricity. The completely commercialised osmotic power plant will be ready by 2015.

The plant generates renewable and emissions-free energy and thus contributes to eco-friendly power production. It is estimated that the osmotic technology will produce 1,600TWh-1,700TWh a year, which is about 13 times the hydropower generated in Norway annually. Out of the total estimated osmotic power of 200TWh that is expected to be generated in Europe, 12TWh will be generated in Norway.

6. Buckingham pi theorem

The pi theorem originated from the French mathematician J. Bertrand in 1978. This technique became widely used due to the works of Rayleigh who used the pi theorem for the first time in it's general aspect.

This theorem is used with the 'similarity principle' which states that systems who differ but share the same description in terms of dimensionless numbers are equivalent.

7. Small scale integration using Buckingham pi theorem.

The power developed through the process is dependent on the following parameters.

$$P = f(A, P_d, \pi_d, Q, A_m, \eta)$$

P = Power produced (Watts).

A = Permeability of the membrane (m²s/kg).

P_d = Pressure of salt water (kg/(m*s²))

π_d = Osmotic Pressure of salt water (kg/(m*s²)).

Q = Flow rate of salt water (m³/s).

A_m = Area of the membrane (m²).

η = Mechanical efficiency of the energy conversion element.

Thus the entire process can be defined a function in the following way:

$$F(P, A, P_d, \pi_d, Q, A_m, \eta) = 0$$

Equation 1 is called as the process equation.

Now,

By using the Buckingham pi theorem for finding the dimensionless π terms we get,

$$\pi_1 = \frac{P_d}{\pi_d}$$

$$\pi_2 = \eta$$

$$\pi_3 = \frac{Q * P_d}{P}$$

$$\pi_4 = \frac{A * P_d^2 * A_m}{P}$$

The principle of similarity says that the values of these terms which would be constants would stay same for all applications for the process equation (eq.1).

The value of this constants for values of parameters used by Statkraft are:

P = 2000 watts.

A = 0.2156 * 10⁻¹⁰ m²s/kg.

P_d = 13 * 10⁵ kg/m*s²

π_d = 26 * 10⁵ kg/m*s².

Q = 0.2 m³/s.

A_m = 2000 m².

η = 0.6~0.7.

{Values are taken from: Fernanda Helfer, Charles Lemckert, Yuri G. Anissimov (Osmotic Power with Pressure Retarded Osmosis: Theory, Performance and Trends – a Review)}

From this terms we get the following relations between the parameters:

$$\pi_1 = \frac{P_d}{\pi_d} = K(\text{generally } 0.5)$$

$$\pi_2 = \eta = 0.6 \sim 0.7$$

$$\pi_3 = \frac{Q * P_d}{P} = 130$$

$$\pi_4 = \frac{A * P_d^2 * A_m}{P} = 36.4346$$

Where,

Flow rate of salt water can also be written as:

$$Q = \frac{P_{in}}{P_d}$$

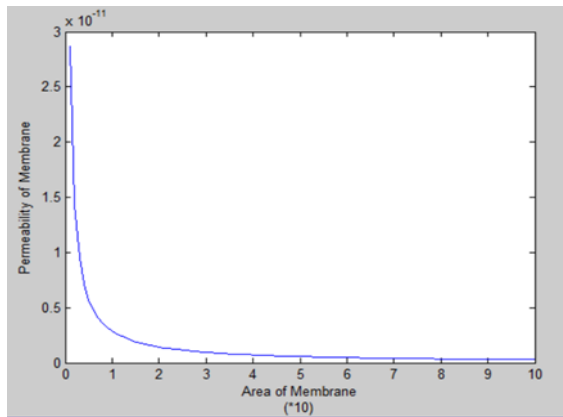
Pin = Input Power to maintain the required discharge (Q) and pressure (Pd).

From the above values we get the following relations between the parameters:

- Relation between A and Am

Equation 1:

$$A * Am = 0.0287 * 10^{-10}$$



This is for Input power of 3000 Watts.

And Output Power of 4000 Watt (for use in a entire home).

If we keep the Membrane Area to be 4m² we get the value of permeability of membrane as:

$$A = 7.175 * 10^{-13} \text{ m}^2\text{/kg}$$

- Relation between P and Pin

$$\frac{P}{(Pin)^2} = 0.01479 * 10^{-3}$$

For,

$$Am = 4 \text{ m}^2,$$

We get,

$$A = 0.2156 * 10^{-10} \text{ m}^2\text{/kg}$$

Next,

$$\frac{P}{(Pin)^2} = 0.05917 * 10^{-3}$$

The above equation is per Area.

Equation 2:

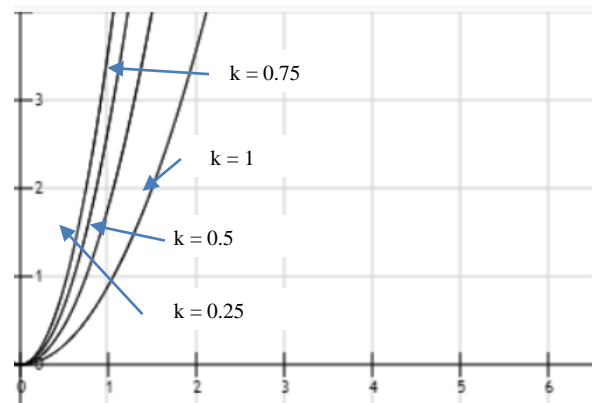
$$\frac{P}{(Pin)^2} = 0.1275 * 10^{-13}$$

The above equation is per Area, per membrane permeability.

- Relation between πd and Pin

Equation 3:

$$\frac{\pi d^2}{Pin} = \frac{0.28}{k^2}$$



8. Generalization

Equations 1, 2 and 3 which are derived above can be used for one’s own respective applications.

It can be typically used for powering a entire house which in general utilizes around 4000 watts of power. It can be seen that for the required power consumption the permeability of the membrane should be 7.175*10⁻¹³ for a membrane area of 4 m².

Conclusion

At present the literature available in application of pressure retarded osmosis is limited as the research though spanned for around two decades is at an emerging stage.

The power plant in Norway by Statkraft is a hydro-power plant which is on very large scale. Through this paper one can use the generalized equations and find the parameters like area required and permeability of the membrane for one’s own use.

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