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Research Article

An overview of Buckling Analysis of Cylinder Subjected to Axially Compressive Load

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Abstract

Buckling is a sudden failure of a structural member subjected to high compressive stress and it is a structural instability leading to a failure mode. Buckling strength of structures depends on many parameters like supports, linear material, composite or nonlinear material etc. This paper intends to study buckling behavior of cylindrical shell which is influenced by thermal loads and imperfections. ANSYS FE non-linear buckling analysis including both material and geometrical non-linearity is used to determine the critical buckling pressure.

Keywords: *Buckling analysis, Boundary conditions, Cylinder shell, Critical buckling pressure, FEA modeling.*

1. Introduction

Cylindrical shells are extensively used in many types of structures which are subjected to various combinations of loading. One of the most critical loads which challenge the stability of thin shells is axial compression. The usual failure mode associated with thin shell structures is buckling. Many investigations have been focusing on the axial compression problem from past 60 years. Love was the first investigator to present a successful linear shell theory based on classical elasticity. Flugge and Byrne presented the second order approximation theory. Donnell developed an eighth order differential equation for determination of critical strength of cylinders with simply supported edges under torsion. Donnell and Flugge highlighted that initial imperfections and the deviation of the actual edge supports from the theoretical support conditions were responsible for observed discrepancy between experimental and theoretical buckling stress values. Batdorf presented a simplified method of elastic stability analysis for thin cylindrical shells. Batdorf, Schildcrout, and Stein employed linear theory as a guide and constructed empirical curves using the data of several of the early investigators. Their Experimentation revealed reduction in critical stress as compared to theoretical values. They highlighted that the observed buckle pattern is different from that predicted on the basis of theory (Himayat Ullah, *et al*, 2007).

When a structure (which is usually subjected to compression) undergoes visibly large displacements transverse to the load then the structure is said to be buckled; for example by pressing the opposite edges of a flat sheet of cardboard towards one another. If a component or part therefore is prone to buckling then its design must satisfy both strength and buckling safety constraints (K. N. Kadam, *et al*, 2013).

Buckling occurs when most of the strain energy which is been stored as membrane energy gets converted into bending energy requiring large deformation resulting in the catastrophic failure (B. Prabu, *et al*, 2009).

2. Mathematical Modeling Approach

Chetan E. Kolambe, *et al*, (2016) gives one of the methods of performing theoretical analysis of thin shell cylinders is given below.

Assumptions are made to perform the calculations:

1. For linear buckling analysis theoretical analysis has been done, internal pressure on the walls is been neglected.
2. Value of factor k=2 for one end fix and other end free condition.
3. Homogeneous material is used throughout shell element.

2.1. Steps for performing the calculations:

1. Calculating critical buckling load:

$$\sigma_{cr} = \frac{\pi^2 * E * t}{k * L^2} \quad (1)$$

Where; E= Modulus of elasticity of Aluminum

I= Moment of Inertia of thin shell = $\frac{\pi}{64} * (D^4 - d^4)$

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L= Length of Shell Element

2. Calculation of Equivalent Stress:

$$\sigma = \frac{2*E*t}{D\sqrt{3(1-v^2)}} \qquad (2)$$

Where, σ= Equivalent stress in GPa
υ = Poison's Ratio = 0.3
D = Outer Diameter of thin shell
E = Modulus of Elasticity
t = Thickness of thin shell in mm

3. Calculation of total deformation

$$\varepsilon = \frac{\delta l}{L} \qquad (3)$$

Where; ε= Elastic Strain
δ= Change in Length or Deformation
L = Original length of thin shell

3. Analytical Modeling

(Himayat Ullah, *et al*, 2007) The analytical model is based on the Kirchhoff–Love hypothesis. The problem is analyzed by the Method of Equilibrium. The three sets of field equations: Equilibrium, Kinematic and Constitutive, along with appropriate boundary conditions comprise the governing equations of the mathematical model.

3.1 Differential Equations of Equilibrium

Pressure Vessels exemplify the axisymmetrically loaded cylindrical shell. Owing to symmetry an element cut from a cylinder of radius 'a' will act on it the internal pressures P_x, P_y, P_z, surface force resultants N_x, N_θ, $N_{\theta x}$, Q_x, Q_θ moment resultants M_x, M_θ , $M_{x\theta}$.

Eliminating the shear forces Q_x and Q_θ and assuming $P_x = P_y = P_z$ $(P_r) = 0$, then the final equilibrium equations are;

$$a\,\partial N_x/\partial x + \partial N_{x\theta}/\partial\theta = 0 \qquad (4)$$

$$\partial N_\theta/d\theta + a\,\partial N_{x\theta}/\partial x + a\ N_x\partial^2 v/\partial x^2 - \partial M_{x\theta}/\partial\theta - \frac{1}{a}\,\partial M_\theta/\partial\theta = 0 \qquad (5)$$

$$a\ N_x\partial^2 w/\partial x^2 + N_\theta + a\ \partial^2 M_x/\partial x^2 + 2\,\partial^2 M_{x\theta}/\partial x\partial\theta + \frac{1}{a}\,\partial^2 M_\theta/\partial\theta^2 = 0 \qquad (6)$$

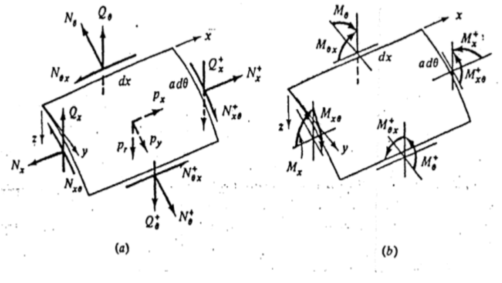

Fig.1 Cylindrical shell element (a) with internal force resultants and surface loads
(b) With internal moment resultants.

3.2 Kinematic Relationships

The strain components at any point through the thickness of the shell are written as;

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\end{bmatrix} = \begin{bmatrix}\varepsilon_{xo}\\ \varepsilon_\theta\\ \gamma_{xyo}\end{bmatrix} - Z\begin{bmatrix}\chi_x\\ \chi_\theta\\ \chi_{x\theta}\end{bmatrix} \qquad (7)$$

Kinematic expressions relating the mid surface strains to the displacement are;

$$\varepsilon_{xo} = \partial u/\partial x \qquad (8)$$
$$\varepsilon_{\theta o} = 1/a\ (\partial v/\partial\theta) - w/a \qquad (9)$$
$$\gamma_{x\theta o} = 1/a\,(\partial u/\partial\theta) + \partial v/\partial x \qquad (10)$$

Similarly the Changes in curvature at any shell point and twist are expressed by;

$$\chi_x = \partial^2 w/\partial x^2 \qquad (11)$$
$$\chi_\theta = 1/a^2\ (\partial x/\partial\theta + \partial^2 w/\partial\theta^2) \qquad (12)$$
$$\chi_{x\theta} = 1/a\ (\partial v/\partial x + \partial^2 w/\partial x\partial\theta) \qquad (13)$$

3.3 Constitutive relations

$$\begin{Bmatrix}\{N\}\\ \{M\}\end{Bmatrix} = \begin{bmatrix}[A] & [0]\\ [0] & [D]\end{bmatrix}\begin{Bmatrix}\{\varepsilon\}\\ \{\chi\}\end{Bmatrix} \qquad (14)$$

Where;
$\{N\} = \{Nx, N\theta, Nx\theta\}^T$ & $\{M\} = \{Mx, M\theta, Mx\theta\}^T$ being the resultant membrane forces and bending moments.

The elastic matrices are given by;

$$[A] = Et/1 - v^2\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & 1-v/2\end{bmatrix} \qquad (15)$$

$$[D] = Et^3/12(1-v^{2)}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & 1-v\end{bmatrix} \qquad (16)$$

3.4 The governing equations for deflection

The expressions governing the deformation of cylindrical shells subjected to direct and bending forces can now be developed. This is accomplished by introducing the compatibility relations into constitutive relations and then subsequently into the equilibrium equations. After differentiation and simplification, the set of differential equations of the buckling problem is as follows:

$$\partial^2 u/\partial x^2 + (1+v)/2a\,\partial^2 v/\partial x\partial\theta - v/a\,\partial w/\partial x + (1-v)/\,2a^2\partial^2 u/\partial\theta^2 = 0 \qquad (17)$$

$$(1+v)/2a\,\partial^2 u/\partial x\partial\theta + (1-v)/2\ \partial^2 v/\partial x^2 + 1\,/\,a^2\ \partial^2 v/\partial\theta^2 - 1/a^2\,\partial w/\partial\theta + \alpha[1/a^2\,\partial^2 v/\partial\theta^2 + 1/a^2\,\partial^3 w/\partial\theta^3\ \ \partial^3 w/\partial x^2\,\partial\theta + (1-v)\,\partial^2 v/\partial x^2] + q\,\partial^2 v/\partial x^2 = 0 \qquad (18)$$

$$aq\,\partial^2\,w/\partial x^2 + v\,\partial u/\partial x + 1/a\ \ \partial v/\partial\theta - w/a - \alpha[1/a\,(\partial^3\,v/\partial\theta^3\,) + a(2-v)\partial^3 v/\partial x^2\partial\theta + a^3\,\partial^4 w/\partial x^4 + 1\,/\,a\ \ \partial^4 w/\partial\theta^4 + 2a\,\partial^4 w/\partial x^2\partial\theta^2] = 0 \qquad (19)$$

Where;

$N_x = N$, $\alpha = t^2/12a^2$ and $q = (1-v^2)\,N/Et$

4. Buckling Analysis- FEA Modeling

The structural static analysis capabilities in the ANSYS program are used to determine the displacements, stresses, strains and forces that occur in a structure or component when the load is been applied to it. Static analysis is an appropriate way to solve problems in which the time dependent effects of inertia and damping don't affect the structure's response. Nonlinearities such as plasticity, creep, large deflection, large strain and contact surfaces are also included in the ANSYS program. A nonlinear static analysis is usually performed by applying the load so as to obtain an accurate solution (K. N. Kadam, *et al*, 2013).

a. Element Type

This analysis considers the shell 63 element which has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. A consistent tangent stiffness matrix option is available for use in large deflection (finite rotation) analyses. Fig.2. shows the details of shell 63 element.

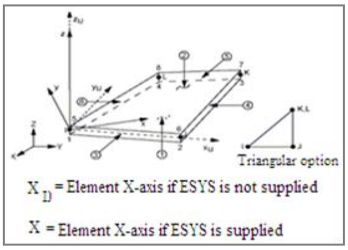

Fig.2 Shows the details of shell 63 element

b. Boundary conditions and modeling of cylinder

For each of the two ends, two different types of boundary conditions (Fig 3) are used. At the fixed end, displacement degrees of freedom in 1, 2, 3 directions (U1, U2, U3) as well as rotational degrees of freedom in 1, 2, 3 directions (θ1, θ2, θ3) were restrained to be zero. At the movable end, load was exerted with an even stress distribution in the longitudinal direction U1 (K. N. Kadam, *et al*, 2013).

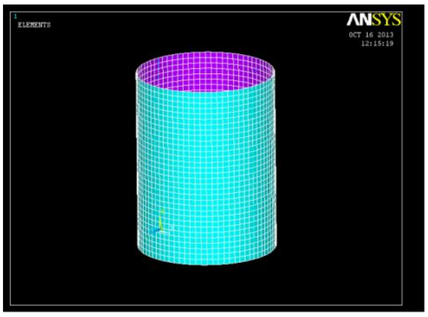

Fig.3 Modeling of cylinder

4.1. Types of Analysis

1. Eigen or linear or bifurcation buckling analysis
2. Non-linear buckling analysis

4.1.1. Eigen Buckling Analysis

B. Prabu, *et al*, (2009) shows Eigen buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure. This analysis is used to predict the bifurcation point using linearized model of elastic structure. It is a technique used to determine buckling pressure-critical pressure-at which a structure becomes unstable and buckled mode shapes-the characteristic shape associated with a structure's buckled response. The other name for this Eigen buckling analysis is "Bifurcation Analysis". The bifurcation buckling refers to unbounded growth of new deformation pattern. This analysis involves calculating the points at which the primary pressure deflection path is bifurcated by a secondary pressure deflection path. ANSYS finite element software package is used to determine the buckling strength of the perfect cylindrical shell through Eigen buckling analysis. In Eigen buckling analysis, imperfections and nonlinearities cannot be included. Sub-space iteration scheme can be used to extract the load factor or Eigen value. The basic form of the Eigen buckling analysis is given by;

$$[K]\{\phi_i\} \equiv \lambda_i [S]\{\phi_i\} \qquad (20)$$

Where;

[K] = Structural stiffness matrix
$\{\phi_i\}$ = Eigen vector
λ_i = Eigen value
{S} = Stress stiffness matrix

K. N. Kadam, *et al*, (2013) has given Eigen value buckling analysis often yields un-conservative results, and should generally not be used in actual day-to-day engineering analyses. The following case of thin-walled cylinders is solved by the ANSYS software.

Table 1 Geometric and Material Properties

Sr. no.	Components	Dimensions
1	Radius	88.2mm
2	Thickness	0.22mm
3	Height	230mm
4	Young's Modulus	205GPa
5	Poison's Ratio	0.3
6	Density	78000N/m^3

Case1: Linear Elastic Buckling Analysis

Fig.4. Shows the deformation Plot of linear elastic Buckling analysis. The maximum deformation is 0.00511mm for the linear Elastic buckling analysis.

Table 2 Comparison of buckling loads of the case studied using Analytical, Numerical and Empirical approach

	Analytical Model	Numerical Models						
		Linear Model	Nonlinear (Perfect)	Buckling Mode Imperfect Geometry			Geometry with Real Imperfection	Empirical Bruhn 90% Probability
Buckling Load (kN)	450	450.1	146.8	GIF	GIF	GIF	81.6	84
				0.50%	5	50		
					%	%		
				114	98	81.6		

Maximum deformation can be observed at the free end and minimum displacement can be observed at the constraint end. The status bar indicates varying displacements across the problem. The Fig.5 shows a buckling stress of around 13.47MPa due to Eigen-Value buckling analysis.

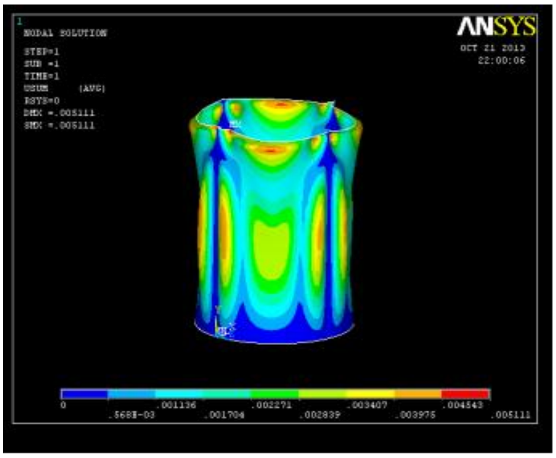

Fig.4 Deformation plot for linear elastic buckling analysis

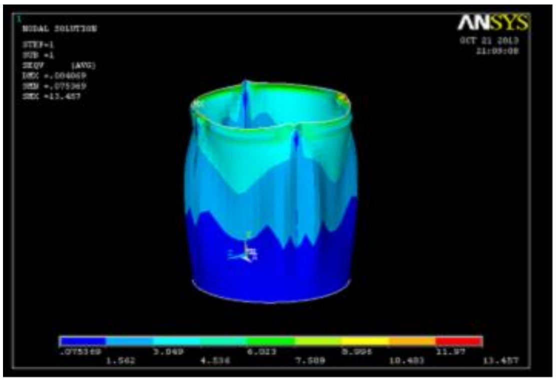

Fig.5 Buckling stress for Eigen-value buckling analysis

4.1.2. Non-linear buckling analysis

Himayat Ullah, *et al*, (2007) predicted Nonlinear buckling analysis is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which a structure becomes unstable. Using the nonlinear technique, we can include features such as initial imperfections, plastic behavior, gaps, and large-deflection response in FE models. To investigate the discrepancy between theoretical and experimental results using numerical technique, the buckling of cylindrical shells is modeled in three ways namely:

1) Non Linear perfect model
2) Buckling mode imperfect geometry
3) Geometry with real imperfection

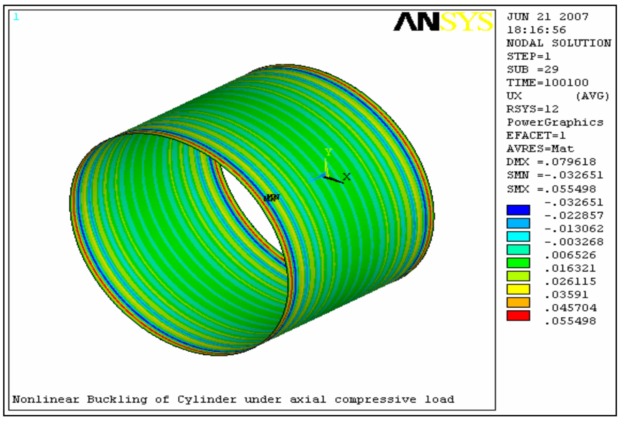

Fig.6 Non Linear Perfect Model (m =13, n=0) (Axisymmetric)

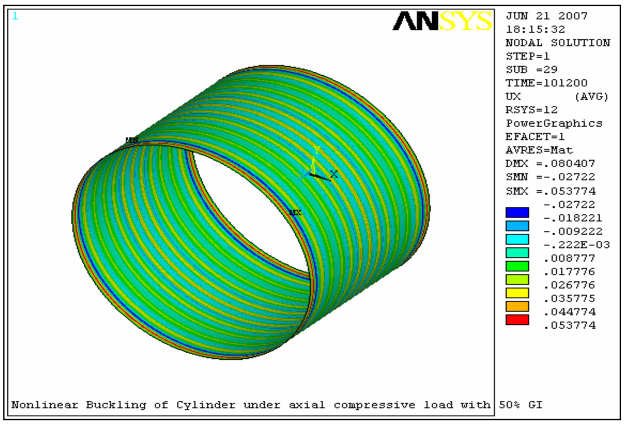

Fig.7 Buckling Mode Imperfect Geometry with 50 % Factor (m =13, n = 0)

Where;
m= Number of waves in longitudinal direction within length of cylinder
n = Number of waves in circumferential direction

Application of theory to the design of actual cylindrical shells is complicated by apparent discrepancies between theory and experiment. This behavior can be seen in comparison of the cylindrical shell results in table 1. The table shows that the analytical and linear numerical model results match very well in case of axial compression.

Conclusions

1. It is observed that, as we go on increasing the number of nodes the total deformation goes on increasing drastically.
2. From case 1 we can say that maximum Deformation is obtained in linear elastic buckling.
3. Deformation, displacement and stress are observed to be maximum at free end rather than constrained end.
4. Buckling mode shape is the same for linear, perfect and imperfect nonlinear models. Linear buckling theory can only be used for determination of mode shapes.
5. Non-linear buckling analysis with geometric imperfections is usually more accurate approach and therefore, it is recommended for design or evaluation of actual cylindrical structures.

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