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Research Article

An α -Series Process Repair Model for A Deteriorating Cold Standby Repairable System with Priority in Repair and Use

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Abstract

This paper studies a deteriorating cold standby repairable system consisting of two dissimilar components and one repairman by assuming that the successive working times form a decreasing α-series process while the consecutive repair times constitute an increasing α-series process, and component 1 has priority in repair and use. A repair replacement policy N based on the number of repairs of component 1 under which the system is replaced when the number of repairs of component 1 reaches N is considered and determined an optimal policy N such that the average cost rate (i.e. The long-run average cost per unit time) of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically.*

Key words: *Geometric process, α-series process, Priority Replacement policy, Renewal reward theorem, Convolution.*

1. Introduction

Many repair replacement models for a one-component repairable system with one repairman are mainly assumed that the system after repair is ‘‘as good as new’’. However, this assumption is not always true in practice applications. Barlow and Hunter first introduced a minimal repair model in which a system after repair has the same failure rate and the same effective age as at the time of failure. Brown and Proschan first investigated an imperfect repair model in which the repair will be perfect with probability p or minimal with probability 1-p. For a deteriorating repairable system, it seems more reasonable to assume that the successive working times of the system after repair will become shorter and shorter while the consecutive repair times of the system after failure will become longer and longer. Ultimately, neither it works nor repaired any more. For such a stochastic phenomenon, Lam first introduced a geometric process repair model to approach it. Under this model, he studied two kinds of replacement policy for a simple repairable system, one based on the working age T of the system and the other based on the failure number N of the system. The explicit equations of the average cost rate under the two policies are derived. Stadje and Zuckerman introduced a general monotone process repair model to generalize Lam's work. Finkelstein presented a general repair model based on a scale transformation after each repair to generalized Lam's work. Zhang combined the two replacement policies used by Lam and proposed a bivariate replacement policy (T, N) under which the system is replaced at the working age T or at the time of the Nth failure, whichever occurs first. Under the same conditions in Lam , he proved that the optimal policy (T,N)* is better than the optimal policies N* and T* . Many replacement policies have been done by Leung and Lee, Wang and Zhang , and others under the geometric process repair model.

In practical applications, some multi-component repairable systems such as series, parallel, standby repairable systems were often installed. Nakagawa and Osaki considered a two-component priority standby redundant system with repair. They assumed that both the working time and the repair time of the priority component have a general distribution while both the working time and the repair time of the non-priority component have an exponential distribution, then some important reliability indices of the system are determined using Markov renewal theory. However, by applying the geometric process repair model, Zhang and Wang [8] investigated a series repairable system with k dissimilar components. They considered a replacement policy M =(N_1, N_2 , ..., N_k) based respectively on the number of failures of component 1, component 2, ... , and component k. An optimal replacement policy M* =(N*1, N*2 , ..., N*k) by minimizing the average cost rate is determined. Moreover, they proved that the optimal policy M* is uniquely existent through theoretical analysis and numerical simulation.

To improve the system reliability, raise system availability or reduce system cost, priority rules are used

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in the multi-component repairable systems. Thus, a standby redundant system consisting of a main component with priority in use or repair and a assistant (standby) component is often installed in practical situations. For example, in the operating room of a hospital, an operation must be discontinued as soon as the power source is cut (i.e. Power station failures). Usually, there is a standby power station (e.g. a storage battery) in the operating room. Thus, the power station (regarded as the main component, e.g Component 1) and the storage battery (regarded as the assistant component,e.g. Component 2) form a cold standby repairable lighting system. Obviously, it is reasonable to assume that the power station has use priority due to the operating cost of the power station is cheaper than the operating cost of the storage battery, and the power station has repair priority due to the used area of the power station is wider than the storage battery (only in the operating room).

Braun et.al [4] examined the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The α-series process grows either as a polynomial or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the α-series process does.

The purpose of this paper is to apply the α-series process repair model to a two-dissimilar-component cold standby repairable system with one repairman and priority unit is not ''as good as new'' after repair and repair times form a increasing α-series process, while, the working times forma decreasing α-series process and component 1 has priority in use and repair. Under these assumptions, we consider a replacement policy N based on the number of repairs of component 1 under which the system is replaced when the repair number of component 1 reaches N. An optimal replacement policy N* is determined such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically.

For understanding the model, we considered the definition (see Braun[4]) of α- series process as follows.

Definition 1:

Given two random variables X and Y, if P(X>t) > P(Y>t) for all real t, then X is called stochastically larger than Y or Y is stochastically less than X. This is denoted by X >st Y or Y <st X respectively.

Definition 2:

Assume that $\{X_n, n=1,2,\ldots\}$, is a sequence of independent non-negative random variables. If the distribution function of X_n is $F_n(t)= F(K^{\alpha}T)$ for some α > 0 and all n=1, 2, 3… then $\{X_n, n=1, 2\ldots\}$ is called an α -series process, α is called exponent of the process.

Obviously:

if α >0, then $\{X_n, n=1,2,\ldots\}$ is stochastically decreasing, i.e, $X_n >_{st} X_{n+1}$, n=1,2,…;

if α <0, then $\{X_n, n=1,2,\ldots\}$ is stochastically increasing, i.e., $X_n <_{st} X_{n+1}$, n=1,2,;

if α =0, then the α -series process becomes a renewal process.

2. Model

We study a two-component cold standby repairable system with one repairman and priority in use and repair by making the following assumptions.

1. In the beginning, the two components are both new, and component 1 is in a working state while component 2 is in a cold standby state.

2. Assume that both components after repair are not ''as good as new'' and follow a α-series process repair. When both components are good, component 1 has the higher use priority than component 2. Even if component 2 is working, it must be switched into the cold standby state as soon as component 1 after failure has been repaired, and it becomes the working state immediately. When both components fail (i.e. the system is down), component 1 has the higher repair priority than component 2. Even if the repairman is repairing component 2 at this stage, he must switch to component 1. He will work on the repair of component 2 after completing the repair on component 1. A possible course of the system is shown in Fig. 1.

3. Assume that $X^{(i)}_n$ and $Y^{(i)}_n$ for i=1,2: n=1,2,3,......are all independent and respectively form a decreasing α –series process and an increasing α –series process. The distribution functions of the working times and repair times forms an exponential failure law.

4. Assume that the time interval between the completion of the (n-1)th repair and the completion of the nth repair of component i is called the nth cycle (i.e. The nth repair cycle) of component i, i=1,2; n=1,2,...

5. Assume that the replacement policy N based on the number of repairs of component 1 is used. The replacement time is negligible. Thus, the system will be replaced by a new and identical one as and when it is down.

6. Assume that any component in the system cannot produce the working reward during cold standby, and no cost is involved during waiting for repair.

7. Assume that the repair cost rate of component 'i' is, C^i_r i=1,2,.. While the working reward rate of two components is same C_w . And the replacement cost of the system is C.

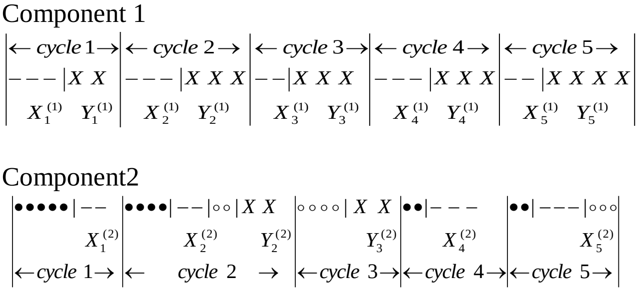


Fig . 1. A possible course of the system -: working state, ● : cold standby state, X: repair state, ○ : waiting for repair state

3. Average Cost Rate under Policy N

In this section, a replacement policy N based on the number of repairs of component 1 is studied. Because the two components appear alternately in the system, when the repair number of component 1 reaches N, then component 1 should work until failure in the (N+1)th cycle whatever component 2 is either in the working state or in the waiting for repair state in the Nth cycle (see Fig. 1). Thus, the renewal point under the policy N is found.

Let τ_1 be the first replacement time of the system, and $\tau_n (n \geq 2)$ be the time between the (n-1)th replacement and the nth replacement of the system under policy N. Obviously $\{ \tau_1, \tau_2, ... \}$ forms a renewal process, and the inter arrival time between two consecutive replacements is called a renewal cycle.

Let C (N) be the average cost rate of the system under policy N. Thus, according to renewal reward theorem (see, for example Ross [2]), we have

$$C(N) = \frac{\text{the expected cost incurred in a renewal cycle}}{\text{the expected length of a renewal cycle}} \tag{3.1}$$

Let W be the length of a renewal cycle of the system under the policy N. Because component 1 has priority in use and repair, component 1 only resides in the working state and the repair state. Thus, based on the enactment of the renewal point under policy N, we have

$$W = \sum_{k=1}^{N+1} X_k^{(1)} + \sum_{k=1}^{N} Y_k^{(1)} \tag{3.2}$$

Where the first term and the second term are, respectively, the length of working time and the length of repair time of component, before the number of repairs of component 1 reaches N.

The total working time 'L' and the total repair time 'R' of the system in a renewal cycle are respectively given by:

$$L = \sum_{k=1}^{N+1} X_k^{(1)} + \sum_{k=1}^{N} X_k^{(2)} I_{\{Y_{k-1}^{(2)} - X_k^{(1)} < 0\}} , \tag{3.3}$$

$$R = R_1 + R_2 = \sum_{k=1}^{N} Y_k^{(1)} + \sum_{k=1}^{N} Y_k^{(2)} I_{\{X_{k-1}^{(2)} - Y_k^{(1)} < 0\}} \tag{3.4}$$

Now, the expectation of W, L and R are respectively given by:

$$E(W) = \sum_{k=1}^{N+1} E\left(X_k^{(1)}\right) + \sum_{k=1}^{N} E\left(Y_k^{(1)}\right) , \tag{3.5}$$

$$E(L) = \sum_{k=1}^{N+1} E\left(X_k^{(1)}\right) + \sum_{k=1}^{N} E\left(X_k^{(2)}\right) E\left(I_{\{Y_{k-1}^{(2)} - X_k^{(1)} < 0\}}\right) , \tag{3.6}$$

$$E(R) = E(R_1) + E(R_2) ,$$

$$= \sum_{k=1}^{N} E\left(Y_k^{(1)}\right) + \sum_{k=1}^{N} E\left(Y_k^{(2)}\right) E\left(I_{\{X_{k-1}^{(2)} - Y_k^{(1)} < 0\}}\right) . \tag{3.7}$$

Where R_1 and R_2 denote, respectively, the repair time of the component 1 and 2 in a renewal cycle, I is the indicator function such that

$$I_A = \begin{cases} 1 \ \text{if event A occurs} \\ 0 \ \text{if event A doesn't occurs} \end{cases}$$

According to assumptions of the model and the definition of the convolution, let the distribution functions of $\left(Y_{k-1}^{(2)} - X_k^{(1)}\right)$ *and* $\left(X_k^{(2)} - Y_k^{(1)}\right)$ are respectively $\boldsymbol{\Phi}_k(u)$ *and* $\boldsymbol{\Psi}_k(v)$.

Where

$$\Phi_k(u) = G^{(2)}\left((k-1)^{\beta_2} u\right) * \left[1 - F^{(1)}\left(-k^{\alpha_1} u\right)\right] , \tag{3.8}$$

$$\Psi_k(v) = F^{(2)}\left(k^{\alpha_2} v\right) * \left[1 - G^{(1)}\left(-k^{\beta_1} v\right)\right] \tag{3.9}$$

and * denotes convolution.

Using the definition of the distribution functions and the concept of the condition distribution, we have

$$\Phi_k(0) = P\left(Y_{k-1}^{(2)} - X_k^{(1)} < 0\right) = \int_0^\infty G_{k-1}^{(2)}(t)\, d F_k^{(1)}(t) , \tag{3.10}$$

$$\boldsymbol{\Psi}_k(\mathbf{0}) = P\left(X_k^{(2)} - Y_k^{(1)} < \mathbf{0}\right) = \int_{\mathbf{0}}^\infty F_k^{(2)}(t)\, d G_k^{(1)}(t) \tag{3.11}$$

Since, it is assumed that $X_k^{(i)}$ *and* $Y_k^{(i)}$, *for* $i = 1,2.$ are all exponential, then their distribution functions are given by

$$F_k^{(i)}(x) = F^{(i)}(k^{\alpha_i} x) = 1 - \exp(-k^{\alpha_i} \lambda_i x), \ \text{for} \ \ i = 1,2.$$

$$G_k^{(i)}(y) = G^{(i)}(k^{\beta_i} y) = 1 - \exp(-k^{\beta_i} \mu_i y), \ \text{for} \ \ i = 1,2. ,$$

$$\text{where}\, x \geq 0, \ y \geq 0, \ 0 \leq \alpha_i \leq 1, \ \ 0 \leq b_i \leq 1, \tag{3.12}$$

$$E\left(Y_k^{(i)}\right) = \int_{\mathbf{0}}^\infty y dG_k^{(i)}(k^{\boldsymbol{\beta}_i} y) = \frac{\mathbf{1}}{\boldsymbol{\mu}_i k^{\boldsymbol{\beta}_i}} , i = \mathbf{1,2}. \tag{3.13}$$

According to the assumptions of the model, definition of probability density function, convolution and Jacobian transformations , the probability density functions of $u = (Y_{k-1}^{(2)} - X_k^{(1)})$ *and* $v = (X_k^{(2)} - Y_k^{(1)})$ are respectively, $\phi_k(u)$ and $\psi_k(v)$

$$\phi_k(u) = \int_0^\infty f(v, u+v)\, dv , \tag{3.14}$$

Where

$$X_k^{(1)} = v, \ \ \mathrm{Y}_{k-1}^{(2)} = u + v, \ \ \text{such that} \ \ u = Y_{k-1}^{(2)} - X_k^{(1)} ,$$

$$\phi_k(u) = \int_0^\infty f(v, u+v)\, dv = \begin{cases} \dfrac{k^{\alpha_1} (k-1)^{\beta_2} \lambda_1 \mu_2}{k^{\alpha_1} \lambda_1 + (k-1)^{\beta_2} \mu_2} e^{-\left(k^{\beta_2} \mu_2 u\right)} & \text{for } u \geq 0 \\ \dfrac{k^{\alpha_1} (k-1)^{\beta_2} \lambda_1 \mu_2}{k^{\alpha_1} \lambda_1 + (k-1)^{\beta_2} \mu_2} e^{-\left(k^{\alpha_1} \lambda_1 u\right)} & \text{for } u < 0 \end{cases} \tag{3.15}$$

$$\psi(v) = \int_0^\infty f(u+v, u)\, du , \tag{3.16}$$

$$\text{where} \quad \mathrm{X}_k^{(2)} = u + v; \ \ \mathrm{Y}_k^{(1)} = u \ \ \text{such that}\ v = X_k^{(2)} - Y_k^{(1)}.$$

$$\psi_k(v) = \int_0^\infty f(u+v, u)\, du$$

$$\psi_k(v) = \int_0^\infty f(u+v, u)\, du = \begin{cases} \dfrac{k^{\alpha_2} k^{\beta_1} \lambda_2 \mu_1}{k^{\alpha_2} \lambda_2 + k^{\beta_1} \mu_1} e^{-\left(k^{\alpha_2} \lambda_2 v\right)} & \text{for } v \geq 0 \\ \dfrac{k^{\alpha_2} k^{\beta_1} \lambda_2 \mu_1}{k^{\alpha_2} \lambda_2 + k^{\beta_1} \mu_1} e^{-\left(k^{\beta_1} \mu_1 v\right)} & \text{for } v < 0 \end{cases} , \tag{3.17}$$

$$E[(Y_{k-1}^{(2)} - X_k^{(1)})\, I_{\left(Y_{k-1}^{(2)}-X_k^{(1)}\right)>0}\,] = \int_0^{\infty} u\,\phi_k(u)\,du\,. \tag{3.18}$$

Using equation (3.15), we have:

$$= \frac{k^{\alpha_1}\lambda_1}{\left(k^{\alpha_1}\lambda_1 + (k-1)^{\beta_2}\mu_2\right)(k-1)^{\beta_2}\mu_2}, \quad k \ge 2 \tag{3.19}$$

$$E[(X_{k-1}^{(2)} - Y_k^{(1)})\, I_{\left(X_{k-1}^{(2)}-Y_k^{(1)}\right)>0}\,] = \int_0^{\infty} v\,\psi_k(v)\,dv \tag{3.20}$$

Let , Using equation (3.17), we have:

$$= \frac{k^{\beta_1}\mu_1}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)k^{\alpha_2}\lambda_2} \quad ,k \ge 1 \tag{3.21}$$

Using equation (3.10), we have:

$$\Phi_k(0) = p(Y_{k-1}^{(2)} - X_k^{(1)} < 0)$$

$$= \frac{(k-1)^{\beta_2}\mu_2}{\left(k^{\alpha_2}\lambda_2 + (k-1)^{\beta_2}\mu_2\right)}, \quad k \ge 2. \tag{3.22}$$

Using equation (3.11), we have:

$$\psi_k(0) = p(X_k^{(2)} - Y_k^{(1)} < 0) = \frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)}, \quad k \ge 1 \tag{3.23}$$

Using equation (3.20), we have:

$$E\left[X_k^{(2)}\, I_{\left(Y_{k-1}^{(2)}-X_k^{(1)}<0\right)}\right] = \frac{1}{\lambda_2 a_2^{k-1}}\Phi_k(0)$$

$$= \frac{1}{\lambda_2 k^{\alpha_2}}\frac{(k-1)^{\beta_2}\mu_2}{\left(k^{\alpha_2}\lambda_2 + (k-1)^{\beta_2}\mu_2\right)} \tag{3.24}$$

Using equation (3.23), we have:

$$E\left[Y_k^{(2)}\, I_{\left(X_k^{(2)}-Y_k^{(1)}<0\right)}\right] = \frac{1}{\mu_2 k^{\beta_2}}\Psi_k(0)$$

$$= \frac{1}{\mu_2 k^{\beta_2}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)} \tag{3.25}$$

By using equation (3.12) and (3.13), equation (3.5) becomes:

$$E(W) = \sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{k=1}^{N}\frac{1}{k^{\beta_1}\mu_1} \tag{3.26}$$

By using equation (3.12) and (3.22), equation (3.6) becomes:

$$E(L) = \sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{(k-1)^{\beta_2}\mu_2}{\left(k^{\alpha_2}\lambda_2 + (k-1)^{\beta_2}\mu_2\right)} \tag{3.27}$$

By using equation (3.13) and (3.23), equation (3.7) becomes:

$$E(R) = \sum_{k=1}^{N}\frac{1}{k^{\beta_1}\mu_1} + \sum_{k=1}^{N}\frac{1}{\mu_2 k^{\beta_2}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)} \tag{3.28}$$

Substituting equations (3.26), (3.27) and (3.28) into equation (3.1), then the av

$$C(N) = \frac{C_r^{(1)}E(R_1) + C_r^{(2)}E(R_2) + C - C_w E(L)}{E(W)} \tag{3.29}$$

$$C(N) = \frac{\left[C_r^{(1)}\sum_{k=1}^{N}\frac{1}{\mu_1 k^{\beta_1}} + C_r^{(2)}\sum_{k=1}^{N}\frac{1}{\mu_2 k^{\beta_1}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)} + C - C_w\left(\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{(k-1)^{\beta_1}\mu_2}{\left(k^{\alpha_2}\lambda_2 + (k-1)^{\beta_1}\mu_2\right)}\right)\right]}{\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}} + \sum_{j=1}^{N}\frac{1}{k^{\beta_1}\mu_1}} \tag{3.30}$$

$$C(N) = \frac{C_r^{(1)}l_2 + C_r^{(2)}l_3 + C - C_w(l_1 + l_4)}{l_1 + l_2} \tag{3.31}$$

$$l_1 = \sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}\ , \qquad l_2 = \sum_{k=1}^{N}\frac{1}{k^{\beta_1}\mu_1}\ ,$$

Where

$$l_3 = \sum_{k=1}^{N}\frac{1}{\mu_2 k^{\beta_2}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2 + k^{\beta_1}\mu_1\right)},$$

and

$$l_4 = \sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{(k-1)^{\beta_2}\mu_2}{\left(k^{\alpha_2}\lambda_2 + (k-1)^{\beta_2}\mu_2\right)}.$$

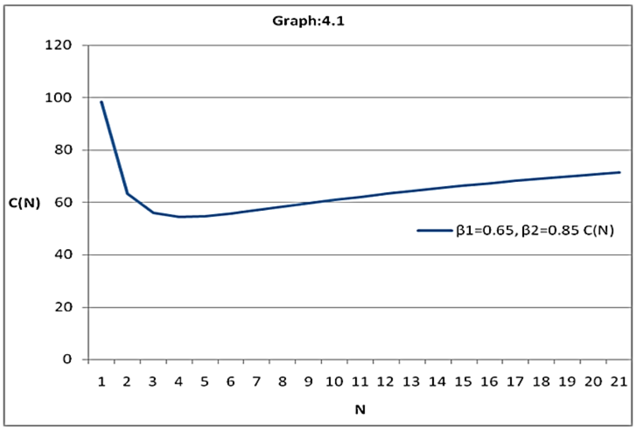
The expression in equation (3.31) gives the long run average cost per unit time. In the next section, it is determine the numerical results for the obtained theoretical results.

4. Numerical Results and Conclusions

For the given hypothetical values of the parameters: $\lambda_1=0.04$, $\lambda_2=0.03$,$\mu_1=0.02$, $\mu_2=0.01$,$\alpha_1=0.65$, $\alpha_2=0.45$, ,$C=6000$, $C_w=10$,$C^{(i)}_r=100$ and $C^{(2)}_r=20$ the numerical results are obtained by the explicit expression given in equation (3.31).

Table 4.1: The values of average cost per unit time

	$\beta_1=0.65$, $\beta_2=0.85$	$\beta_1=0.55$, $\beta_2=0.75$
N	C(N)	C(N)
1	98.45333	98.45333
2	63.40048	64.23246
3	55.93858	56.72156
4	54.3658	54.98588
5	54.66673	55.1261
6	55.65576	55.97721
7	56.89854	57.10555
8	58.21609	58.32865
9	59.53005	59.56434
10	60.80581	60.77473
11	62.02892	61.94278
12	63.19468	63.06173
13	64.30309	64.13001
14	65.35637	65.1486
15	66.35778	66.11979
16	67.31086	67.04637
17	68.21922	67.93131
18	69.08628	68.77756
19	69.91523	69.58791
20	70.70901	70.36499
21	71.47032	71.1112



Conclusions

1. From the table 4.1 and graph 4.1, it is examined that the long-run average cost per unit time is minimum when the number of failure of the component 1 reaches 4.i.e., C(4)=56.3658 at β1=0.65, β2=0.85.Thus the system should be replaced at the time of 4th failure.
2. The decreasing α -series process may be more appropriate for modelling system working times while the increasing α-series process is more suitable for modelling repair times of the system. It assumed that each component 1 after repair is not 'as good as new' and also the successive working times form a decreasing α -series process, the successive repair time's form an increasing α-series process and both the processes are exposing to exponential failure law and component 1 has priority in use and repair. Under these assumptions we study an optimal replacement policy N in which we replace the system when the number of failures of component 1 reaches N. We determine an optimal repair replacement policy N* such that the long run average cost per unit time is minimized.
3. If the repair man is familiar with the repair the successive repair times stochastically non-increasing and the successive working times are stochastically non-decreasing. Thus this model can also be applied for an improved system exposing to Weibull failure law.

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