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Research Article

Non-Linear Mixed-Oberbeck Elecrtoconvection in a Poorly Conducting Fluid Through a Vertical Channel in the Presence of Electric Field

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Abstract

Nonlinear Mixed –Oberbeck electroconvection in a poorly conducting fluid in a vertical channel in the presence of an electric field, viscous and Ohmic dissipations is investigated. The flow is governed by the modified Navier-Stokes and energy equations with boundaries which are kept at different temperatures. The nonlinear coupled momentum and energy equations are solved numerically using finite difference technique and analytically using regular perturbation method with B_r as the perturbation parameter. The velocity and temperature fields are obtained for various values of electric number, Brinkman number and GR which is nothing but the ratio of Grashof number to the Reynolds number. We note that , if GR=0 ,the problem reduces to the forced convection. We have computed the local Nusselt number at the walls, the skin friction and mass flow rate. The analytical solutions are compared with those obtained from the numerical solution and good agreement is found. We found the effect of increase in the temperature difference between the plates is to increase the velocity and temperature distributions due to increase in convection. These results are useful in the effective control of heat transfer in many industrial problems.

Keywords*: Mixed Oberbeck Convection, Poorly Conducting Fluid, Skin friction, Heat transfer, Mass flow rate, Nusselt Number, Grashof Number. Brinkmann Number, Perturbation method and Electric Field.*

1. Introduction

The study of combined free and forced convection called mixed convection(MC) in a vertical channel has received considerable attention because of its wide range of applications from cooling of electronic devices ,gas-cooled nuclear reactors to that of solar energy collectors. A comprehensive review on mixed convection can be found in and. One of the earliest study on laminar, fully developed mixed convection in a vertical channel with uniform wall temperature was by Tao[3].Recently, Aung and Worku,,Cheng et al, have studied the mixed convection in a vertical channel with symmetric and asymmetric heating at the walls. These authors reported that the buoyancy force can cause flow reversal both for upward and downward flows. More recently Barlcua , has studied the fully developed MC flow in a vertical channel with viscous dissipation. An analytical solution is found by a perturbation method. In particular he has studied the, forced convective flow with viscous heating as the base heat transfer process while the effect of buoyancy is accounted for by expressing the fluid velocity and temperature as power series in the ratio between the Grashof number and Reynolds number. Aung and Worku , discussed the theory of combined free and forced convection in a vertical channel with flow reversal conditions for both developing and fully developed flows. Rudraiah et al [8] have studied the effect of oscillatory flow on mixed convection in a vertical porous stratum.

In contrast to MC due to variation of density with temperature discussed above there is another type of MC arising in a poorly conducting alloys like nickle-titanium(N_i-T_i), aluminum oxides and so on in a vertical channel due to variation of electrical conductivity, σ ,with temperature in the presence of an electric field, $\vec{E}$,called mixed electroconvection (MEC).The variations of σ with temperature releases the charges forming distribution of charge density, ρ_e (See).These charges in turn induces the electric field, $\vec{E}_i$ called thermal or induced electric field. In addition these may be an applied electric field, $\vec{E}_a$,due to embedded electrodes of different potentials at the boundaries (See Fig.1).The total electric field $\vec{E}(=\vec{E}_i+\vec{E}_a)$ in a poorly conducting fluid produces a current which acts as sensing. In addition this $\vec{E}$ together with ρ_e produces an electric force $\rho_e\vec{E}$ which acts as

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actuation. These two properties are the two important properties required a material to be a smart material. Therefore MEC plays a significant role to synthesis smart materials like shape memory alloys(SMA).These smart materials demonstrate great potential for enhancing the functionality, serviceability and durability of civil, mechanical, electrical and electronic infrastructure systems. Is spite of these importance much attention has not been given to the study of MEC in a vertical channel.The study of it in the presence of total electric field, viscous and Joulean dissipations is the objective of this paper.

To achieve this objective we first obtain the modified conservation of momentum, energy together with Maxwell equations and continuity of charges, suitable for a poorly conducting mixed oberbeck electro convection. Modified in the sense of considering electric force in the momentum equation and ohmic dissipation in the energy equation.

2 Mathematical Formulation

Consider a steady, laminar, fully developed mixed convective flow in an open-ended vertical parallel plate channel filled with poorly conducting fluid. The Oberbeck-Boussinesq approximation is assumed to be valid following Rajagopal et al (1996). In other words we consider Oberbeck-Boussinesq, homogeneous, poorly conducting fluid, together with electrohydrodynamic (EHD) approximations(See Rudraiah et al,1996), namely σ is very small and hence induced magnetic field is negligible and there is no applied magnetic field.

It is assumed that the thermal conductivity, the dynamic viscosity and the thermal expansion coefficient are considered constant. The X-axis is chosen parallel to the gravitational field, but with opposite direction and Y-axis is transverse to the plates. The origin is such that the channel walls are at positions Y=0 and Y=2L, respectively as shown in figure 1.

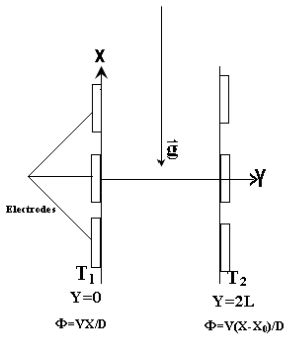

Fig 1: Physical Configuration

By the conduction of fully developed flow, the mass balance equation will be

$\frac{\partial U}{\partial X}=0$,so that U depends only on Y, where u is the velocity along the X-axis.

Then the required basic equations for steady flow are

$$g\beta(T-T_0)-\frac{1}{\rho_0}\frac{\partial P}{\partial X}+\frac{\mu}{\rho_0}\frac{d^2U}{dY^2}+\frac{\rho_e E_x}{\rho_0}=0 \quad (2.1)$$

$$\chi\frac{d^2T}{dY^2}+\frac{\mu}{(\rho c_p)_f}\left(\frac{dU}{dY}\right)^2+\frac{\sigma|\vec{E}|^2}{(\rho cp)_f}=0 \quad (2.2)$$

$$\sigma\left(\frac{d^2\varphi}{dX^2}+\frac{d^2\varphi}{dY^2}\right)+\frac{d\varphi}{dY}.\frac{d\sigma}{dY}=0 \quad (2.3)$$

and the Maxwell equations for a poorly conducting fluid are

$\nabla.\vec{E}=\frac{\rho_e}{\varepsilon_0},\quad \vec{E}=-\nabla\phi,\quad \vec{J}=\sigma\vec{E}$, $\sigma=\sigma_0\left[1+\alpha_b(T-T_0)\right]$ (2.4a,b,c,d)

$\sigma=1+\alpha\theta_B$ Where $\alpha=\alpha_b\Delta T,$ (2.4e)

We assume that the temperature of the boundary at Y=0 is T_1.while that at Y=2L is $T_2$with $T_2\geq T_1$ and dP/dX is independent of X. Let there exist a constant A such that

$$\frac{dP}{dX}=A \quad (2.5)$$

Evaluating the derivative of equation (2.1) with respect to X and using equation (2.5), we obtain

$$\frac{dT}{dX}=0 \quad (2.6)$$

This implies that the temperature also depends only on the variable Y. The energy balance equation includes the effects of viscous and ohmic dissipations.

Equation (2.1), using (2.2) and (2.5), becomes

$$\frac{d^4U}{dY^4}-\frac{g\beta}{\chi(c_p)_f}\left(\frac{dU}{dY}\right)^2-\frac{g\beta}{\nu K}\sigma|\vec{E}|^2+\frac{1}{\mu}\frac{d^2}{dY^2}(\rho_e E_x)=0 \quad (2.7)$$

3 Boundary Conditions

The no-slip boundary co nditions on U are

$$U(0)=U(2L)=0 \quad (3.1)$$

$$T=T_1 \quad at \quad Y=0$$

$$T=T_2 \quad at \quad Y=2L \quad (3.2)$$

Using Equations (2.1) and (2.4),and using the boundary conditions on T we obtain

$$\left(\frac{d^2U}{dY^2}\right)_{Y=0}=\frac{A}{\mu}-\frac{g\beta(T_1-T_0)}{\nu}-\frac{\rho_e E_x}{\mu} \quad (3.3a)$$

$$\left(\frac{d^2U}{dY^2}\right)_{Y=2L}=\frac{A}{\mu}-\frac{g\beta(T_2-T_0)}{\nu}-\frac{\rho_e E_x}{\mu} \quad (3.3b)$$

$$\phi=VX/D \quad at \quad Y=0$$

$$\phi=V(X-X_0)/D \quad at \quad Y=2L \quad (3.4)$$

Equations (2.1)-(2.4) and (3.1)–(3.4) can be written in a dimensionless form by employing the dimensionless quantities

$$u^*=\frac{U}{U_0},\theta=\frac{T-T_0}{\Delta T},y^*=\frac{Y}{D},E^*=\frac{E}{V/D},x^*=\frac{X}{D},\rho_e^*=\frac{\rho_e}{\varepsilon_0 V/D^2},\phi^*=\frac{\phi}{V}, \quad (3.5)$$

where D=2L is the hydraulic diameter. The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0 = -\frac{AD^2}{48\mu}; \qquad T_0 = \frac{T_1+T_2}{2} \tag{3.6}$$

The physical meanings of other quantities defined in (6.3.5) are explained in the earlier chapters. Moreover, the reference temperature difference ΔT is given by

$$\Delta T = T_2 - T_1 \text{ if } T_1 < T_2 \tag{3.7}$$

Using (3.5), equations (2.1)-(2.4) and (3.1)-(3.4) become,

$$\frac{d^2\theta}{dy^2} + B_r\left(\frac{du}{dy}\right)^2 + T_e\sigma\left[E_x^2 + E_y^2\right] = 0 \tag{3.8}$$

$$\frac{d^4u}{dy^4} - GR.B_r\left(\frac{du}{dy}\right)^2 - GR.T_e\sigma\left[E_x^2 + E_y^2\right] + W_e\frac{d^2}{dy^2}(\rho_e E_x) = 0 \tag{3.9}$$

$$\sigma\left(\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2}\right) + \frac{d\varphi}{dy}.\frac{d\sigma}{dy} = 0 \tag{3.10}$$

$$\nabla.\vec{E} = \rho_e,\ \sigma = 1+\alpha\theta \quad \text{Where } \alpha = \alpha_b\Delta T, \tag{3.11}$$

$$u(0) = u(1) = 0 \tag{3.12}$$

$$\theta = -1/2 \quad at \quad y = 0$$
$$\theta = 1/2 \quad at \quad y = 1 \tag{3.13}$$

$$\left(\frac{d^2u}{dy^2}\right)_{y=0} = -48 + \frac{GR}{2} - W_1 \tag{3.14}$$

$$\left(\frac{d^2u}{dy^2}\right)_{y=1} = -48 - \frac{GR}{2} - W_1e^{-\alpha} \tag{3.15}$$

$$\phi = x \quad at \quad y = 0$$
$$\phi = (x - x_0) \quad at \quad y = 1 \tag{3.16}$$

Further (2.1) using scales (3.5) and (3.6) can be written as

$$\frac{d^2u}{dy^2} + GR\theta + W_e(\rho_e E_x) + 48 = 0 \tag{3.17}$$

Rearranging (3.17), we get

$$\theta = -\frac{1}{GR}\left(48 + \frac{d^2u}{dy^2}\right) - \frac{W_e}{GR}(\rho_e E_x) \tag{3.18}$$

where

$$Gr = \frac{g\beta\Delta TD^3}{\nu^2}, R_e = \frac{U_0D}{\nu}, B_r = \frac{\mu{U_0}^2}{K\Delta T}, GR = \frac{Gr}{\mathrm{Re}}\ W_e = \frac{\varepsilon_0V^2}{\rho_0g\beta\Delta TD^3}$$

4 **Solution for ϕ**:

The solution for ϕ, according to (3.10), depends on σ which in turn depends on the temperature θ as in (3.11). In a poorly conducting fluid (i.e.,σ<<1), the dissipations in (2.2) are negligible and hence σ will depend on the conduction temperature, θ_B,satisfying

$$\frac{d^2\theta_B}{dy^2} = 0 \tag{4.1}$$

The solution of this satisfying the boundary conditions

$$\theta_B = -1/2 \quad at \quad y = 0 \ and\ \theta_B = 1/2\ at \quad y = 1 \tag{4.2}$$

is $$\theta_B = y - 1/2 \tag{4.3}$$

Then (3.11) becomes

$$\sigma = 1+\alpha(y-1/2) \approx e^{\alpha(y-1/2)}\ (\because \alpha << 1) \tag{4.4}$$

Then (3.11) using (4.4) becomes

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \alpha\frac{\partial\phi}{\partial y} = 0 \tag{4.5}$$

To find the solution of (4.5), as in the previous chapters, we consider the following two cases

Case 1: The Potential Difference Applied Opposite to the Temperature Difference

The solution of (4.5), satisfying (3.16), is

$$\phi = x - \frac{x_0}{(1-e^{-\alpha})}(1-e^{-\alpha y}) \tag{4.6}$$

From (2.4a, b, c, d), after making dimensionless using the quantities defined earlier and using (4.6), we get

$$\rho_e = \nabla.\vec{E} = -\nabla^2\phi = -\frac{\alpha^2x_0}{(1-e^{-\alpha})}e^{-\alpha y},\ E_x = -1,\ E_y = \frac{\alpha x_0}{(1-e^{-\alpha})}e^{-\alpha y} \tag{4.7}$$

Also $$\rho_eE_x = \frac{\alpha^2x_0}{(1-e^{-\alpha})}e^{-\alpha y} \quad {E^2}_x + {E^2}_y = 1 + \frac{{x_0}^2\alpha^2e^{-2\alpha y}}{(1-e^{-\alpha})^2}, \tag{4.8}$$

Case 2. The Potential Difference Applied in the Same Direction of Temperature Difference

In this case the boundary conditions on ϕ, in dimensionless form, are opposite to those specified in (3.16) and they are

$$\phi = x \quad at \quad y = 1 \quad and \quad \phi = x - x_0 \quad at \quad y = 0 \tag{4.9}$$

In this case the solution of (4.5), satisfying (4.9),is

$$\phi = x + \frac{x_0}{(1-e^{-\alpha})}(e^{-\alpha} - e^{-\alpha y}) \tag{4.10}$$

We note that ρ_e, E_y and ρ_eE_x are opposite to those obtained in case 1, where as E_x and ${E_x}^2 + {E_y}^2$ remain the same as in case 1.

5 Analytical Solution

Since (3.8) and (3.17) are coupled equations, we find the solutions of these equations analytically using perturbation technique with Brinkman number (Br) as the perturbation parameter, using the boundary conditions given in (3.12) and (3.13) for different temperature combinations mentioned below. Another way of finding the solution of the problem is to use (3.9) along with the boundary conditions (3.12),(3.14) and (3.15).We observe that in both the cases , we will get the same values. In this section we find the solution below following the first method. We obtain that, when the buoyancy forces are dominating i.e., when $GR \to \pm\infty$,in the case of asymmetric heating, it is purely natural convection. On the other hand, when buoyancy forces are negligible and viscous dissipation is dominating, i.e., GR=0 , a purely forced convection occurs.The solution of (3.8) and (3.17) for a fixed value of GR≠0 is obtained, for small values of B_r , using the series

$$u(y) = u_0(y) + B_ru_1(y) + {B_r}^2u_2(y) + \ldots = \sum_{n=0}^{\infty} {B_r}^n u_n(y) \tag{5.1}$$

$$\theta(y) = \theta_0(y) + B_r\theta_1(y) + {B_r}^2\theta_2(y) + \ldots = \sum_{n=0}^{\infty} {B_r}^n\theta_n(y) \tag{5.2}$$

Case 1: Isothermal-Isothermal Walls

Substituting (5.1) and (5.2) in (3.8) and (3.17), we get

$$\frac{d^2\theta}{dy^2}+B_r\left(\frac{du}{dy}\right)^2+T_e e^{-\alpha/2}e^{\alpha y}+W_2e^{-\alpha y}=0 \tag{5.3}$$

$$\frac{d^2u}{dy^2}+GR\theta+W_1e^{-\alpha y}+48=0 \tag{5.4}$$

Substituting series (5.1)and (5.2) in to (5.3),(5.4)and equating coefficients of like powers of B_r to zero, we obtain the boundary value problem for n=0 and 1 as

$$\frac{d^2u_0}{dy^2}+GR\theta_0+W_1e^{-\alpha y}+48=0 \tag{5.5}$$

$$\frac{d^2\theta_0}{dy^2}+T_e e^{-\alpha/2}e^{\alpha y}+W_2e^{-\alpha y}=0 \tag{5.6}$$

$$\frac{d^2u_1}{dy^2}+GR\theta_1=0 \tag{5.7}$$

$$\frac{d^2\theta_1}{dy^2}+B_r\left(\frac{du_0}{dy}\right)^2=0 \tag{5.8}$$

$$u_0(0)=u_0(1)=0 \tag{5.9}$$

$$u_1(0)=u_1(1)=0 \tag{5.10}$$

$$\theta_0=-1/2 \quad at \quad y=0 \; and \; \theta_0=1/2 \; at \quad y=1 \tag{5.11}$$

$$\theta_1=0 \quad at \quad y=0 \; and \; 1 \tag{5.12}$$

Solving (6.5.5) to (6.5.8) using (6.5.9) to (6.5.12), we get

$$\theta_0=-W_2\left(e^{-\alpha}-e^{-\alpha y}-1\right)+T_e e^{-\alpha/2}\left(e^{\alpha}-e^{-\alpha y}-1\right)+\alpha^2 y \tag{5.13}$$

$$u_0=\frac{a_1}{\alpha^2}\left(y+e^{-\alpha y}-e^{-\alpha}-1\right)+\frac{a_2}{\alpha^2}\left(y+e^{\alpha y}-e^{\alpha}-1\right)+\frac{GRc_1}{6}+\frac{a_3}{2}\left(1-y^2\right) \tag{5.14}$$

$$\theta_1=-\left(\frac{a_1^2e^{-2\alpha y}+a_2^2e^{2\alpha y}}{4\alpha^4}\right)+y^2\left(\frac{a_2GRC_1}{\alpha^3}e^{\alpha y}-\frac{a_1GRC_1}{\alpha^3}e^{-\alpha y}\right)+\left(a_{12}e^{\alpha y}++a_{13}e^{-\alpha y}\right)$$
$$+y\left(C_5+\frac{a_5}{2}y-\frac{a_3C_3}{3}y^2+\frac{a_4}{4}y^3+\frac{GRC_1a_3}{20}y^4-\frac{GRC_1^2}{120}y^5+a_{10}e^{\alpha y}+a_{11}e^{-\alpha y}\right)+C_6 \tag{6.5.15}$$

$$u_1=\left(a_{18}e^{-2\alpha y}+a_{19}e^{2\alpha y}\right)++y\left(a_{27}e^{\alpha y}+a_{28}e^{-\alpha y}+C_7\right)+\left(a_{29}e^{\alpha y}+a_{30}e^{-\alpha y}\right)+C_8$$
$$+y^2\left(a_{25}e^{\alpha y}+a_{26}e^{-\alpha y}-\frac{GRC_6}{2}-\frac{C_5GR}{6}y+a_{24}y^2+a_{23}y^3+a_{22}y^4+a_{21}y^5+a_{20}y^6\right) \tag{5.16}$$

Here, $W_i\left(i=1\,to\,2\right), C_i\left(i=1\,to\,8\right), a_i\left(i=1\,to\,30\right)$,

$$W_1=\frac{W_e x_0\alpha^2}{\left(1-e^{-\alpha}\right)}, \quad W_2=\frac{T_e x_0^2\alpha^2e^{-\alpha/2}}{4\left(1-e^{-\alpha}\right)^2}$$

are constants.

Case2: Isothermal-Isoflux Walls

The Isoflux and Isothermal boundary conditions for the channel walls are given by

$$q_1=K\left(\frac{dT}{dY}\right)_{Y=0} \quad ; \qquad T(2L)=T_2 \tag{5.17}$$

In the non-dimensional form of (5.17), using (3.5) with $\Delta T=\frac{q_1D}{K}$,becomes

$$\left(\frac{d\theta}{dY}\right)_{y=0}=1 \quad ;\theta(1)=\theta_2 \tag{5.18}$$

where $\theta_2=\frac{T_2-T_0}{\Delta T}$ is the thermal ratio parameter

Using (5.18) in (4.1) we get

$$\sigma=1+\alpha\left(y+\Omega\right)\approx e^{\alpha(y+\Omega)}, \tag{5.19}$$

where $\Omega=\theta_2-1$

Using this, we get

$$\phi=x-\frac{x_0}{\left(1-e^{-\alpha}\right)}\left(1-e^{-\alpha y}\right) \tag{5.20}$$

From (5.20) we obtain $\rho_e E_x, E_x, E_y$.

Following the above procedure, (3.8) and (3.17) take the form

$$\frac{d^2u}{dy^2}+GR\theta+W_1e^{-\alpha y}+48=0 \tag{5.21}$$

$$\frac{d^2\theta}{dy^2}+B_r\left(\frac{du}{dy}\right)^2+T_e\,e^{\alpha\Omega}e^{\alpha y}+W_4e^{-\alpha y}=0 \tag{5.22}$$

Substituting series (5.1)and(5.2) in to (5.21)and(5.22)and equating coefficients of like powers of B_r to zero, we obtain the boundary value problem for n=0 and 1 as

$$\frac{d^2u_0}{dy^2}+GR\theta_0+W_1e^{-\alpha y}+48=0 \tag{5.23}$$

$$\frac{d^2\theta_0}{dy^2}+T_e e^{\alpha\Omega}e^{\alpha y}+W_4e^{-\alpha y}=0 \tag{5.24}$$

$$\frac{d^2u_1}{dy^2}+GR\theta_1=0 \tag{5.25}$$

$$\frac{d^2\theta_1}{dy^2}+B_r\left(\frac{du_0}{dy}\right)^2=0 \tag{5.26}$$

$$u_0(0)=u_0(1)=0 \tag{5.27}$$

$$u_1(0)=u_1(1)=0 \tag{5.28}$$

$$\left(\frac{d\theta_0}{dy}\right)_{y=0}=1 \; and \; \theta_0=1/2 \; at \quad y=1 \tag{5.29}$$

$$\left(\frac{d\theta_1}{dy}\right)_{y=0}=0 \; and \; \theta_1=0 \; at \quad y=1 \tag{5.30}$$

Solving (5.23) to (5.26) using (5.27) to (5.30), we get

$$\theta_0=2T_e e^{\alpha\Omega}\left(e^{\alpha}-e^{\alpha y}+\alpha\left(y-1\right)\right)$$
$$+2W_4\left(e^{-\alpha}-e^{-\alpha y}+\alpha\left(1-y\right)\right)+\alpha^2\left(2y-1\right) \tag{5.31}$$

$$u_0=\frac{a_1}{\alpha^2}\left(y+e^{-\alpha y}-e^{-\alpha}-1\right)+\frac{a_2}{\alpha^2}\left(y+e^{\alpha y}-e^{\alpha}-1\right)+\frac{GRC_1}{6}+\frac{a_3}{2}\left(1-y^2\right) \tag{5.32}$$

$$\theta_1=-\left(\frac{a_1^2e^{-2\alpha y}+a_2^2e^{2\alpha y}}{4\alpha^4}\right)+y\left(a_{10}e^{\alpha y}+a_{11}e^{-\alpha y}+C_5\right)+\left(a_{12}e^{\alpha y}+a_{13}e^{-\alpha y}+C_6\right)$$
$$+y^2\left(\frac{a_2GRC_1}{\alpha^3}e^{\alpha y}-\frac{a_1GRC_1}{\alpha^3}e^{-\alpha y}+\frac{a_5}{2}-\frac{a_3C_3}{3}y+\frac{a_4}{4}+\frac{GRC_1a_3}{20}y^3-\frac{GRC_1^2}{120}y^4\right) \tag{5.33}$$

$$u_1=\left(a_{18}e^{-2\alpha y}+a_{19}e^{2\alpha y}\right)+\left(a_{29}e^{\alpha y}+a_{30}e^{-\alpha y}\right)+y\left(-\frac{C_5GR}{6}y^2-\frac{GRC_6}{2}y+C_7\right)$$
$$+y^2\left(a_{25}e^{\alpha y}+a_{26}e^{-\alpha y}+a_{24}y^2+a_{23}y^3+a_{22}y^4+a_{21}y^5+a_{20}y^6\right)+C_8 \tag{5.34}$$

Where $W_i\left(i=1,4\right), C_i\left(i=1\,to\,8\right), a_i\left(i=1\,to\,30\right)$,

$$W_4=\frac{T_e x_0^2\alpha^2e^{\alpha\Omega}}{4\left(1-e^{-\alpha}\right)^2}$$

are constants.

Case3: Isoflux-Isoflux Walls

The Isoflux and Isoflux boundary conditions for the channel walls are given by

$$q_1 = K\left(\frac{dT}{dY}\right)_{Y=0} \quad ; \qquad q_2 = K\left(\frac{dT}{dY}\right)_{Y=1} \tag{5.35}$$

In the non-dimensional form of (5.35), using (3.5) with $\Delta T = \frac{q_1 D}{K}$,becomes

$$\left(\frac{d\theta}{dY}\right)_{y=0} = 1 \quad ;\left(\frac{d\theta}{dY}\right)_{y=1} = \lambda_1 \tag{5.36}$$

In addition to this we use the boundary condition is

$$\int_0^1 \theta dy = Q \tag{5.37}$$

where $\lambda_1 = \frac{q_2}{q_1}$ is the ratio of heat fluxes.

Using (5.36)and (5.37)in (4.1) and then from(3.11) we get

$$\sigma = 1 + \alpha\left(\lambda_1 y + \Omega_1\right) \approx e^{\alpha\left(\lambda_1 y + \Omega_1\right)} \tag{5.38}$$

Using this in (3.10) and then solving using(3.16),we get

$$\phi = x - \frac{x_0}{\left(1 - e^{-\alpha_1}\right)}\left(1 - e^{-\alpha_1 y}\right) \tag{5.39}$$

where $\Omega_1 = Q - \frac{\lambda_1}{2}, \alpha_1 = \alpha\lambda_1$ is constant.

From (5.39) we obtain $\rho_e E_x, E_x, E_y$.

Following the above procedure, we get the following equations

$$\frac{d^2 u_0}{dy^2} + GR\theta_0 + W_1 e^{-\alpha_1 y} + 48 = 0 \tag{5.40}$$

$$\frac{d^2\theta}{dy^2} + B_r\left(\frac{du}{dy}\right)^2 + T_e\, e^{\alpha\Omega_1} e^{\alpha_1 y} + W_3 e^{-\alpha_1 y} = 0 \tag{5.41}$$

Substituting series(5.1)and(5.2) in to (5.40)and(5.41)and equating coefficients of like powers of B_r to zero, one obtains the boundary value problem for n=0 and 1 as

$$\frac{d^2 u_0}{dy^2} + GR\theta_0 + W_1 e^{-\alpha_1 y} + 48 = 0 \tag{5.42}$$

$$\frac{d^2\theta_0}{dy^2} + T_e\, e^{\alpha\Omega_1} e^{\alpha_1 y} + W_3 e^{-\alpha_1 y} = 0 \tag{5.43}$$

$$\frac{d^2 u_1}{dy^2} + GR\theta_1 = 0 \tag{5.44}$$

$$\frac{d^2\theta_1}{dy^2} + B_r\left(\frac{du_0}{dy}\right)^2 = 0 \tag{5.45}$$

$$u_0(0) = u_0(1) = 0 \tag{5.46}$$

$$u_1(0) = u_1(1) = 0 \tag{5.47}$$

$$\left(\frac{d\theta_0}{dY}\right)_{y=1} = \lambda_1 \text{ and } \int_0^1 \theta_0 dy = Q \tag{5.48}$$

$$\left(\frac{d\theta_1}{dY}\right)_{y=1} = 0 \text{ and } \int_0^1 \theta_1 dy = 0 \tag{5.49}$$

Solving (5.42) to (5.45) using (5.46) to (5.49), we get

$$\theta_0 = \frac{T_e e^{\alpha\Omega_1}}{2\alpha_1^3}\left(2e^{\alpha_1} y\alpha_1 - e^{\alpha_1}\alpha_1^2 - 2e^{\alpha_1 y}\alpha_1 + 2\left(e^{\alpha_1} - 1\right)\right)$$
$$+ \frac{W_3}{2\alpha_1^3}\left(e^{\alpha_1}\alpha_1^2 - 2e^{\alpha_1 y}\alpha_1 - 2e^{\alpha_1} y\alpha_1 + 2W_3\left(1 - e^{-\alpha_1}\right)\right) + \frac{\lambda_1}{2}(2y-1) + Q \tag{5.50}$$

$$u_0 = \frac{l_1}{\alpha_1^2}\left(y + e^{-\alpha_1 y} - e^{-\alpha_1} - 1\right) + \frac{l_2}{\alpha_1^2}\left(y + e^{\alpha_1 y} - e^{\alpha_1} - 1\right) + \frac{GRd_1}{6} + \frac{l_3}{2}\left(1 - y^2\right) \tag{5.51}$$

$$\theta_1 = -\left(\frac{l_1^2 e^{-2\alpha_1 y} + l_2^2 e^{2\alpha_1 y}}{4\alpha_1^4}\right) + y\left(l_{10} e^{-\alpha_1 y} + l_{11} e^{\alpha_1 y} + d_5\right) + \left(l_{12} e^{-\alpha_1 y} + l_{13} e^{\alpha_1 y} + d_6\right)$$
$$y^2\left(-\frac{l_2 GRd_1 e^{-\alpha_1 y} + l_1 GRd_1 e^{\alpha_1 y}}{\alpha_1^3}\right) + y^2\left(\frac{l_4}{4}y^2 - \frac{l_3 d_3}{3}y + \frac{l_5}{2} + \frac{GRd_1 l_3}{20}y^3 + -\frac{GR^2 d_1^2}{120}y^4\right) \tag{5.52}$$

$$u_1 = \left(\frac{GRl_1^2 e^{2\alpha_1 y} + GRl_2^2 e^{-2\alpha_1 y}}{16\alpha_1^6}\right) + y^2\left(\frac{l_2 GR^2 d_1 e^{-\alpha_1 y} - l_1 GR^2 d_1 e^{\alpha_1 y}}{\alpha_1^5}\right)$$
$$+ y\left(l_{22} e^{-\alpha_1 y} + l_{23} e^{\alpha_1 y} + d_7 - \frac{d_6 GR}{2}y\right) + \left(l_{24} e^{-\alpha_1 y} + l_{25} e^{\alpha_1 y} + d_8\right)$$
$$+ y^3\left(-\frac{d_5 GR}{6} - -\frac{l_5 GR}{24}y + \frac{d_3 l_3 GR}{60}y^2 - \frac{l_4 GR}{120}y^3 - \frac{GR^2 d_1 l_3}{120*7}y^4 + \frac{GR^3 d_1^2}{120*56}y^4\right) \tag{5.53}$$

where $W_i\,(i = 1,3), d_i\,(i = 1\,to\,8), l_i\,(i = 1\,to\,25)$,

$$W_3 = \frac{T_e x_0^2 \alpha_1^2 e^{\alpha\Omega_1}}{4\left(1 - e^{-\alpha_1}\right)^2}$$

are constants.

6 Numerical Method

The analytical solutions obtained in Section 5 using regular perturbation technique are valid only for small values of the parameter B_r. However in many practical problems like unconventional generators, shock tubes, nuclear reactors and so on, the values of B_r. are usually large and regular perturbation solutions cannot be used. For arbitrary values of B_r., analytical solutions for the nonlinear coupled equations derived in section 5 are complicated and hence as in the previous chapters, we resort to numerical solution using the finite difference technique. For numerical method, we consider the first case. The velocity and energy equations are solved using the central difference method. The use of central difference scheme replaces the derivative with the corresponding central difference approximations leading to the set of linear algebraic equations. The solutions of the reduced algebraic equations are obtained by the Method of Relaxation .The relaxation parameter ω is fixed by comparing the numerical results with those obtained by analytical method. The convergence criterion is based on the step size and the previous iterations for the iterative difference to the order 10^{-6}.

6.1 Isothermal-Isothermal Walls

The finite difference equations of (5.3) and (5.4) using central difference scheme with 21 mesh point with the step size (h) 0.05 are:

$$\frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} + W_1 e^{-\alpha y_j} + GR\theta_j + 48 = 0 \quad , \tag{6.1.1}$$

Solving for u_j

$$L_1 = u_j = \frac{1}{2}\left[u_{j+1} + u_{j-1} + W_1 e^{-\alpha y_j} h^2 + GR\theta_j h^2 + 48h^2\right] \tag{6.1.2}$$

Similarly,

$$\frac{\theta_{j+1}-2\theta_j+\theta_{j-1}}{h^2}+Br\left(\frac{u_{j+1}-u_{j-1}}{2h}\right)^2+T_e e^{-\alpha/2}e^{\alpha y_j}+W_2 e^{-\alpha y_j}=0 \quad (6.1.3)$$

Solving for θ_j

$$\theta_j=\frac{1}{2}\left[\theta_{j+1}+\theta_{j-1}+\frac{Br}{4}\left(u_{j+1}-u_{j-1}\right)^2+T_e h^2 e^{-\alpha/2}e^{\alpha y_j}+W_2 h^2 e^{-\alpha y_j}\right] \quad (6.1.4)$$

where h = (Δy) = 0.05 and j represents the mesh point and it varies from 0 to 20.

Equations (6.1.3) and (6.1.4) give two implicit equations in u_j and θ_j denoted by L_1 and L_2 respectively. They separately generate systems of linear equations in u_j, u_{j+1}, u_{j-1} and $\theta_j, \theta_{j+1}, \theta_{j-1}$, resulting in tri-diagonal co-efficient matrices. These equations are solved using the Method of Relaxation.

The Algorithm

Initially we choose all u_j and θ_j to be zero. This assumption is automatically corrected as the iteration proceeds. The Scheme of the Relaxation Method for the k^{th} iteration and j^{th} mesh point is:

$u_j^k=\omega[u_j^k]_{L_1}+(1-\omega)u_j^{k-1}$ and $\theta_j^k=\omega[\theta_j^k]_{L_2}+(1-\omega)\theta_j^{k-1}$

Here ω is the relaxation parameter.

This translates into:

$$u_j^k=\omega\left[\frac{1}{2}\left[u_{j+1}^k+u_{j-1}^k+W_1 e^{-\alpha y_j}h^2+GR\theta_j h^2+48h^2\right]\right]+(1-\omega)u_j^{k-1} \quad (6.1.5)$$

and similarly for θ we get:

$$\theta_j^k=\omega\left[\frac{1}{2}\left[\theta_{j+1}^k+\theta_{j-1}^k+\frac{Br}{4}\left(u_{j+1}-u_{j-1}\right)^2+h^2T_e e^{-\alpha/2}e^{\alpha y_j}+W_2 h^2 e^{-\alpha y_j}\right]\right]+(1-\omega)\theta_j^{k-1} \quad (6.1.6)$$

where $y_j=y_0+jh$ with an initial guess and y_0 = 0.

The boundary conditions for u, given by $u_{y=0}=0$ and $u_{y=1}=0$, are incorporated as:

u_0 =0 and u_{20} =0

Similarly, we use $\theta_{y=0}=-0.5$ and $\theta_{y=1}=0.5$ .That is $\theta_0=-0.5$ and $\theta_{20}=0.5$

Equations (6.1.5) and (6.1.6) are solved simultaneously from k=0 at each mesh *j* point till the required order of iterative difference is achieved after comparison with the analytical results.

6.2 Case2: Isothermal-Isoflux Walls

Even in this case, applying the same procedures as described above to (5.21) and (.22), we get:

$$u_j^k=\omega\left[\frac{1}{2}\left[u_{j+1}^k+u_{j-1}^k+W_1 e^{-\alpha y_j}h^2+GR\theta_j h^2+48h^2\right]\right]+(1-\omega)u_j^{k-1} \quad (6.2.1)$$

The boundary conditions for u given by $u_{y=0}=0$ and $u_{y=1}=0$ are incorporated as:

u_0 =0 and u_{20} =0.

Since we encounter a Neumann Boundary Condition at one wall for θ, we have employed the Shooting Method to determine θ_j from j=0 to 20. The boundary conditions for θ are given by $\frac{d\theta}{dy}_{y=0}=1$ and $\theta_{y=1}$ = 0.5. (6.2.2)

We define: $\left(\frac{d\theta}{dy}\right)_j=t_j$, and make an assumption $\theta_0=1$. (6.2.3)

Therefore (6.6.22) becomes

$$\left(\frac{dt}{dy}\right)_j+Br\left(\frac{u_{j+1}-u_{j-1}}{2h}\right)^2+T_e e^{\alpha\Omega}e^{\alpha y j}+W_4 e^{-\alpha y j}=0 \quad (6.2.4)$$

, $t_0=0$

Equations (6.2.3) and (6.2.4) convert the boundary value problem (5.22) into two initial value problems in θ and *t* which are solved using the Euler's Method as given below:

$$\theta_j^k=\theta_{j-1}^k+h\left(\frac{d\theta}{dy}\right)_{j-1} \quad (6.2.5)$$

$$t_j^k=t_{j-1}^k+h\left(\frac{dt}{dy}\right)_{j-1} \quad (6.2.6)$$

Where $\frac{d\theta}{dy}$ and $\frac{dt}{dy}$ are given by (6.2.3) and (6.2.4) respectively. The equation (6.2.1) is separately solved at each mesh point *j* for k=0. Then equations (6.2.3)-(6.2.6) are solved simultaneously at each mesh point *j* for k=0. This process is then iterated and in the end θ_0 is varied so that we get $\theta_{y=1}$ =0.5.

6.3 Case3: Isoflux-Isoflux Walls

Applying the same procedure described above to (5.40) we get:

$$u_j^k=\omega\left[\frac{1}{2}\left[u_{j+1}^k+u_{j-1}^k+W_1 e^{-\alpha y_j}h^2+GR\theta_j h^2+48h^2\right]\right]+(1-\omega)u_j^{k-1} \quad (6.3.3)$$

The boundary conditions for u given by $u_{y=0}=0$ and $u_{y=1}=0$ are incorporated as:

u_0 =0 and u_{20} =0.

Since we encounter Neumann Boundary Condition at both the walls for θ, we have employed the Shooting Method to determine θ_j for j=0 to 20. The boundary conditions for θ given are given by $\frac{d\theta}{dy}_{y=0}=1$ and $\frac{d\theta}{dy}_{y=1}=\lambda_1$. Proceeding in the same way as described in Section 6.2, to (5.41) we get :

$\left(\frac{d\theta}{dy}\right)_j = t_j$, assuming $\theta_0 = 1$.

(6.3.4)

$$\left(\frac{dt}{dy}\right)_j + Br\left(\frac{u_{j+1}-u_{j-1}}{2h}\right)^2 + T_e e^{\alpha\Omega_1} e^{\alpha_1 y_j} + W_3 e^{-\alpha_1 y_j} = 0 \quad , \quad t_0 = 0$$

(6.3.5)

$$\theta_j^k = \theta_{j-1}^k + h\left(\frac{d\theta}{dy}\right)_{j-1} \qquad (6.3.6)$$

$$t_j^k = t_{j-1}^k + h\left(\frac{dt}{dy}\right)_{j-1} \qquad (6.3.7)$$

The above equations are solved as described in Section 6.2 and the value of θ_0 is varied so to get $\left(\frac{d\theta}{dy}\right)_{y=1}=\lambda_1$.

These solutions for *u* and θ in each section are computed for different values of the parameters and the results are depicted graphically in figures (6.2) to (6.16) along with the analytical results and conclusions are drawn.

7 Skin friction, rate of heat transfer and mass flow rate

We found the Skin friction, Rate of heat transfer and Mass flow rate using the expressions after non dimensionless

$\tau' = \mu\left(\partial U/\partial Y\right)_{Y=0,2L}$, $q' = -K\left(\partial T/\partial y\right)_{y=0,2L}$,

$$m_E = \int_{-b}^{b} \rho_0 u dy$$

8. Results and discussions

The effect of electric field on the velocity, temperature, are obtained both analytically and numerically and the results are depicted in figures. The analytical solution for mixed convective poorly conducting fluid flow and heat transfer in a vertical channel is obtained using regular perturbation method. A numerical procedure is also performed to obtain the solutions in the presence of dissipative forces. The viscous and ohmic dissipations terms are included in the energy equation. The analytical solutions have been determined using Br as perturbation parameter. The heat transfer coefficient is obtained for three different thermal boundary conditions.

The velocity and temperature fields in the case of asymmetric heating are obtained and are depicted in figs 6.2a to 6.3b,(6.7a) to (6.8b) and(6.12a) to(6.13b).For asymmetric heating the temperature specified at the boundaries are different and hence velocity and temperature fields depend on the perturbation parameter Br. Figs 6.2c, 6.7b ,6.12.c show that when the flow is upward GR is positive. On the other hand when the flow is downward, GR is negative. It is also clear that analytical and numerical solutions agree very well. This is due to the fact that for GR>0 there is a buoyancy assisted flow and hence flow is upward and for GR<0 there is buoyancy opposed flow and hence flow is downward. We can also observe that the velocity increases with an increase in the value of GR.A greater energy generated by dissipation forces yields greater fluid temperature which in turn increases the buoyancy force.

The figures 6.2b, 6.7b, 6.12 represent velocity for different values of electric number W_e and for all the three types of thermal boundary conditions. When temperature difference (ΔT)is in the same direction of potential difference ($\Delta\varphi$).We find the electric field is to decreases the velocity with an increases in W_e . This implies that the effect of electric field is to suppress the convection .On the other hand, if $\Delta\varphi$ and ΔT are in the opposite direction, its augments convection.

Figs 6.2d,6.7d,6.12d gives the velocity for different values of B_r. As Br increases the velocity also increases and flow reversal occurs for 2nd and 3rd cases. Figs 6.3a,b,,6.8a,b,6.13a,b, represent the temperature profiles for different values of W_e and T_e. In case 1, the increase in W_e and T_e increase the heat transfer. In case 2, as W_e and T_e increase, the heat transfer also increases.

Figs 6.4a,b, 6.9a,b, 6.14a,b, give the skin friction for different values of B_r . Skin friction decreases linearly through the distance y and also skin friction decreases with an increase in B_r.

Similarly ,figs 6.5a,b,,6.10a,b,6.15a,b, show that the rate of heat transfer decreases with an increase in B_r .Also figures 6.6a,b,,6.11a,b,6.16a,b, show that the mass flow rate increases with the increase in W_e and B_r for all the three different thermal boundary conditions. Finally, we conclude that the dissipations enhance the effect of flow reversal in the case of downward flow while it encounter this effect in the case of upward flow.

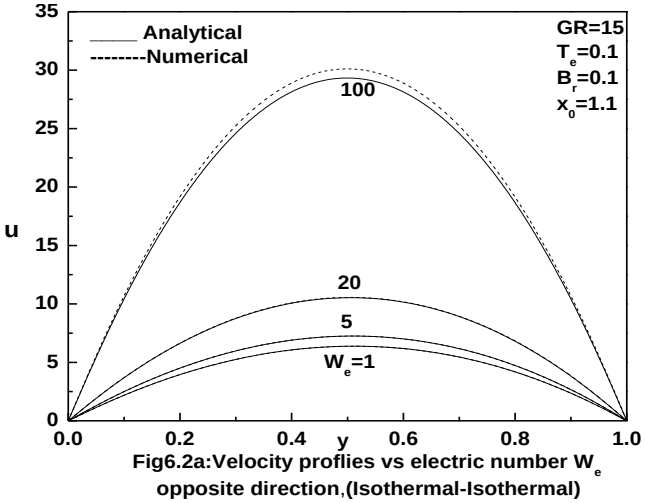

Fig6.2a:Velocity proflies vs electric number W_e opposite direction,(Isothermal-Isothermal)

Finally we conclude that with a proper choice of external constraints of electric field and the temperature difference it is possible to control MOBEC, which is useful in the manufacture of new materials like smart materials free from impurities.

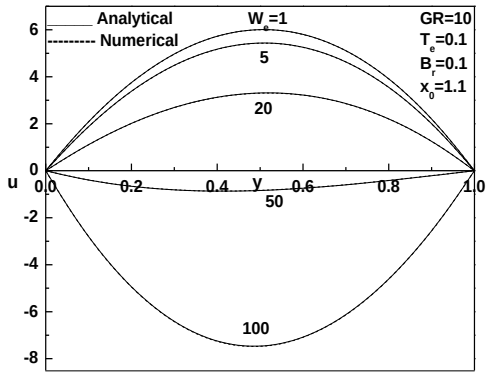

Fig6.2b:Velocity proflies vs electric number W_e (same direction),(Isothermal-Isothermal)

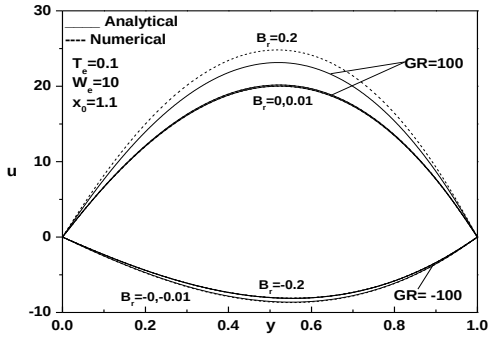

Fig6.2c: Velocity profiles vs B_r (Isothermal-Isothermal)

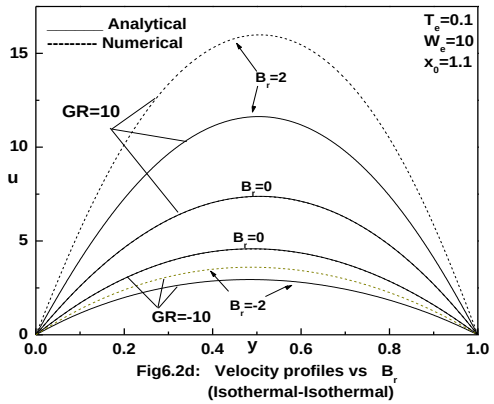

Fig6.2d: Velocity profiles vs B_r (Isothermal-Isothermal)

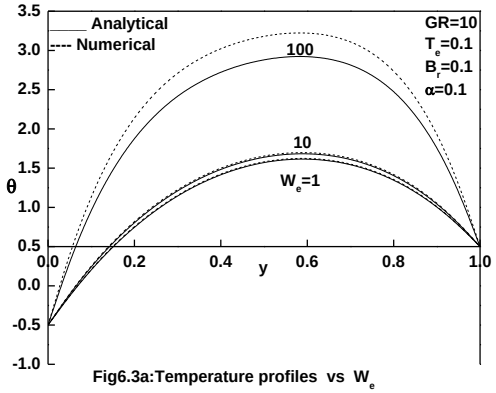

Fig6.3a:Temperature profiles vs W_e (Isothermal-Isothermal)

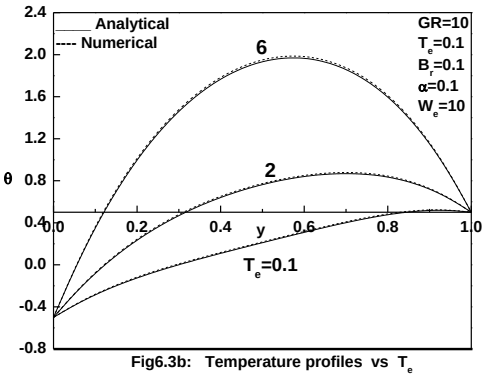

Fig6.3b: Temperature profiles vs T_e (Isothermal-Isothermal)

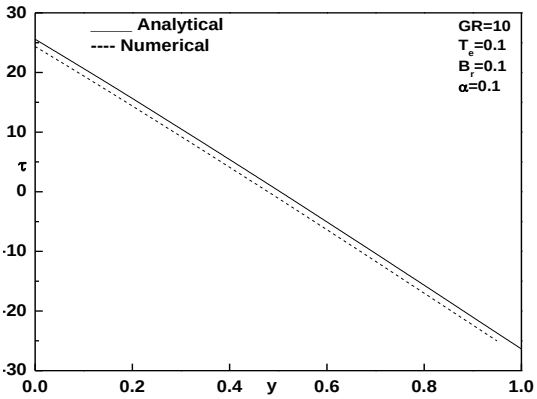

Fig6.4a: Skin friction vs Y with different W_e (Isothermal-Isothermal)

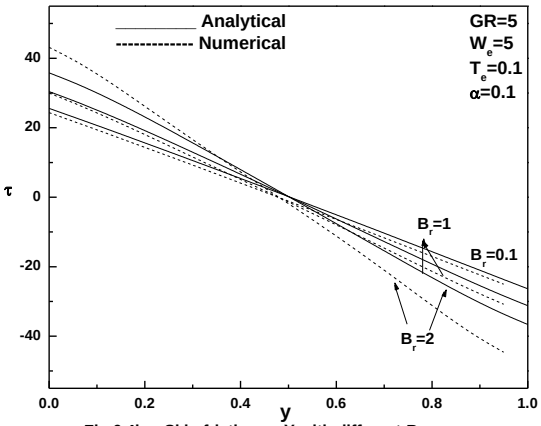

Fig 6.4b: Skin friction vs Y with different B_r (Isothermal-Isothermal)

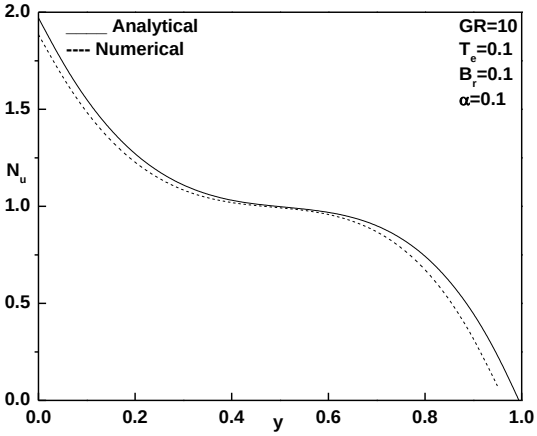

Fig6.5a: Heat transfer vs Y with different W_e (Isothermal-Isothermal)

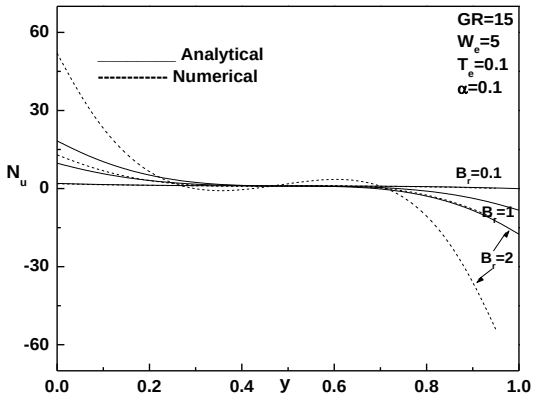

Fig6.5b: Heat transfer with Y with different B_r (Isothermal-Isothermal)

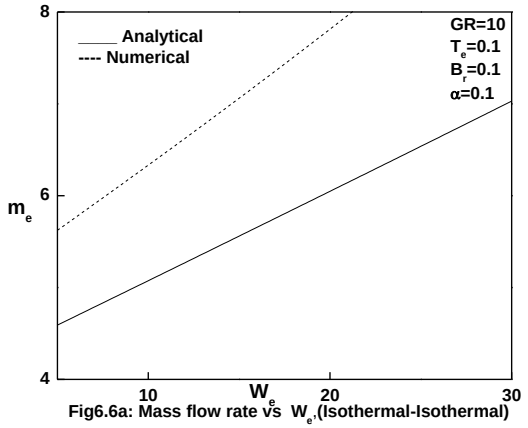

Fig6.6a: Mass flow rate vs W_e,(Isothermal-Isothermal)

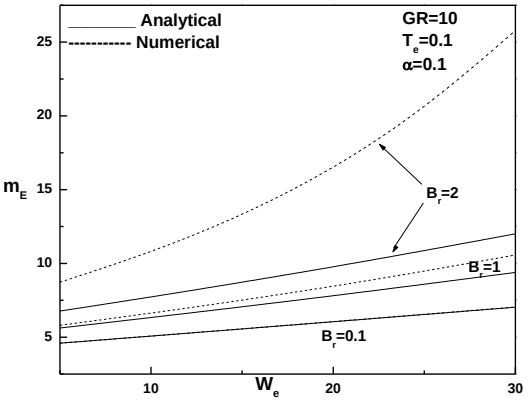

Fig6.6b: Mass flow rate vs W_e with different B_r (Isothermal-Isothermal)

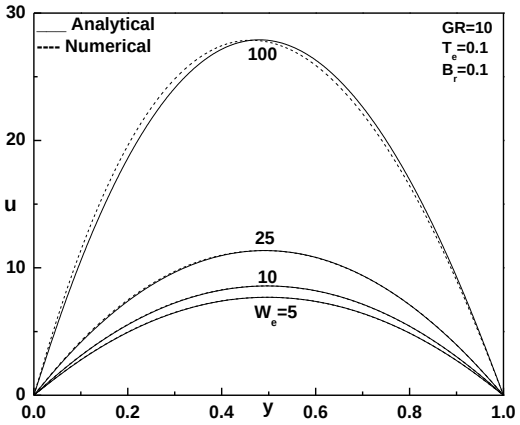

Fig6.7a:Velocity profiles vs Y for different values W_e (Opposite direction),(Isothermal-Isoflux)

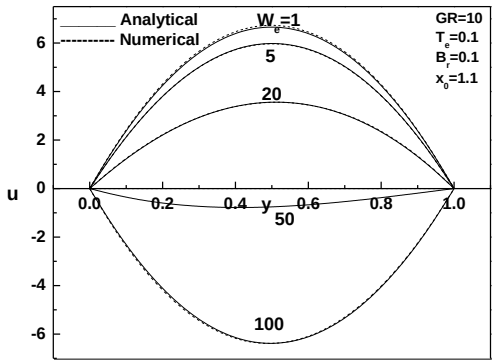

Fig6.7b: Velocity proflies vs Y for differnt values of W_e (Same direction),(Isothermal-Isoflux)

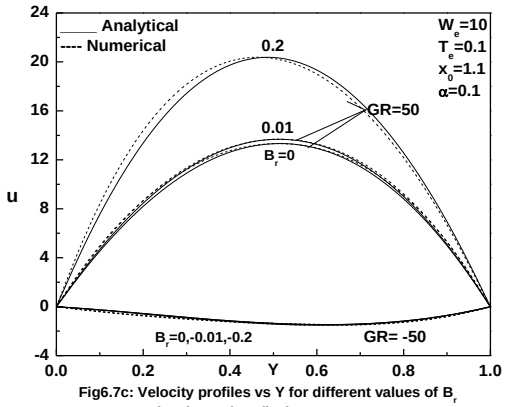

Fig6.7c: Velocity profiles vs Y for different values of B_r (Isothermal-Isoflux)

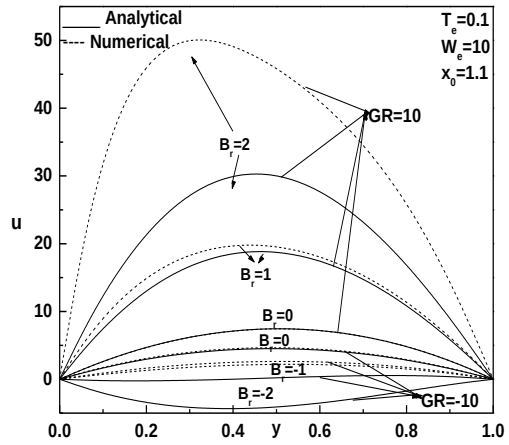

Fig 6.7d: Velocity profiles vs Y for different values of B_r (Isothermal-Isoflux)

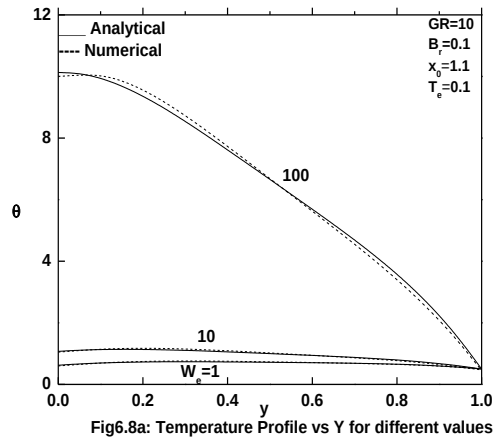

Fig6.8a: Temperature Profile vs Y for different values of W_e (Isothermal-Isoflux Walls)

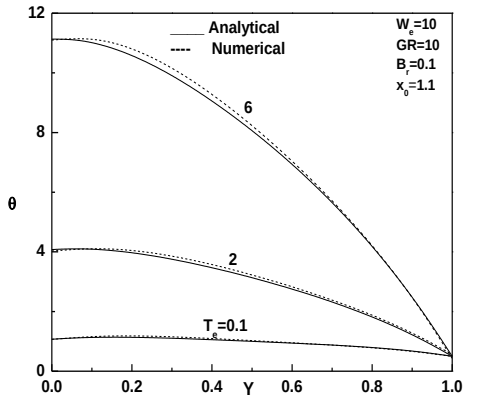

Fig6.8b: Temperature Profile vs Y different values of T_e
(Isothermal-Isoflux Walls)

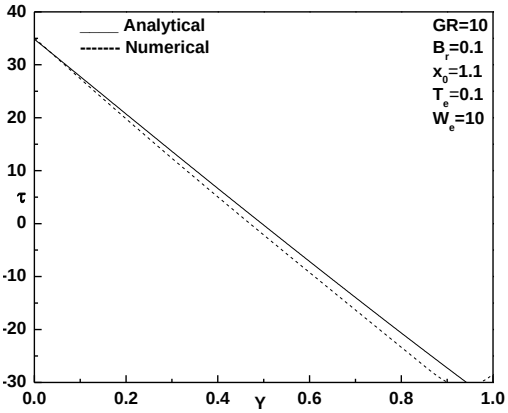

Fig6.9a: Skin friction vs Y
(Isothermal-Isoflux Walls)

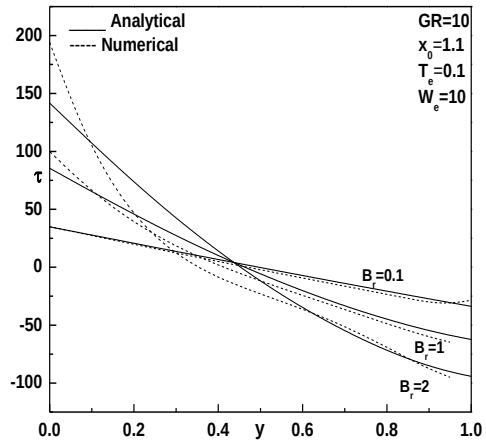

Fig6.9b: Skin friction vs Y for different values of B_r
(Isothermal-Isoflux Walls)

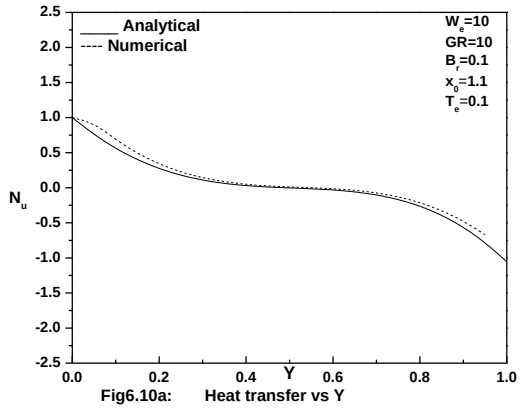

Fig6.10a: Heat transfer vs Y
(Isothermal-Isoflux Walls)

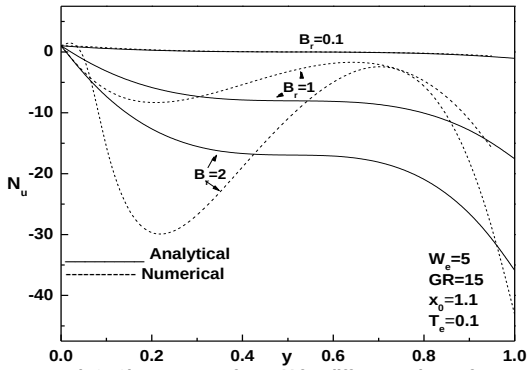

Fig6.10b: Heat transfer vs Y for different values of B_r
(Isothermal-Isoflux Walls)

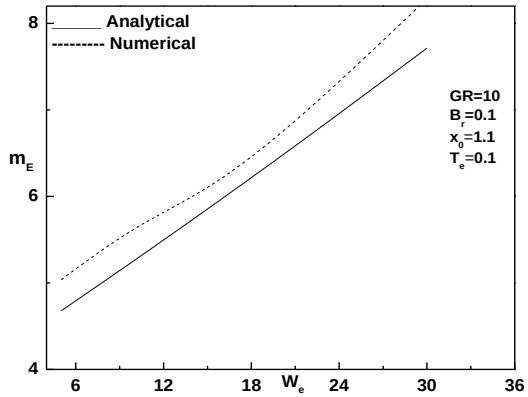

Fig 6.11a: Mass flow rate vs W_e
(Isothermal-Isoflux Walls)

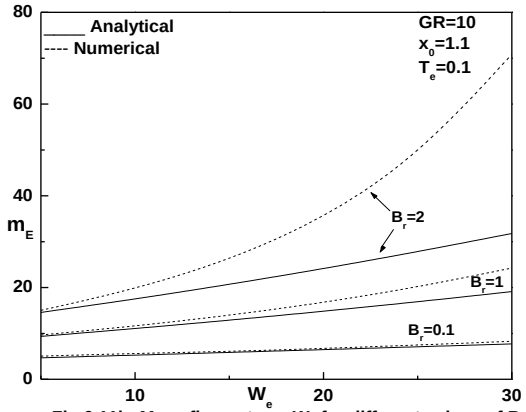

Fig 6.11b: Mass flow rate vs W_e for different values of B_r
(Isothermal-Isoflux Walls)

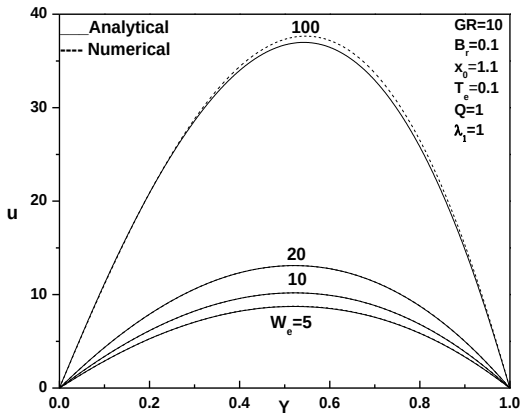

Fig 6.12a:Velocity Profile vs Ydifferent values of W_e
(Isoflux-Isoflux Walls)

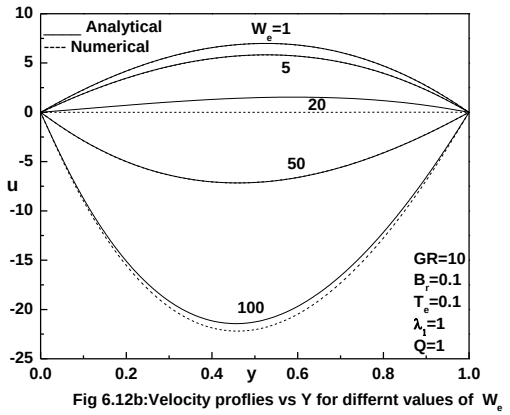

Fig 6.12b:Velocity profiles vs Y for differnt values of W_e (Same direction),(Isoflux-Isoflux)

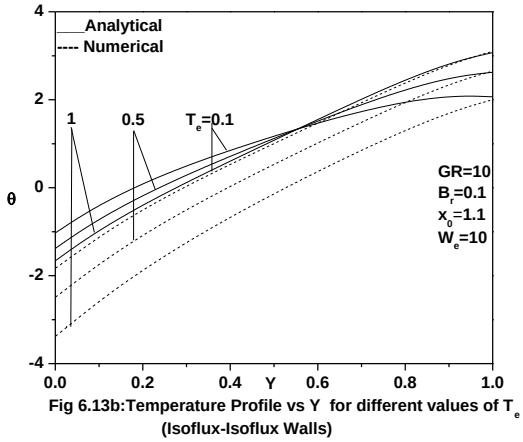

Fig 6.13b:Temperature Profile vs Y for different values of T_e (Isoflux-Isoflux Walls)

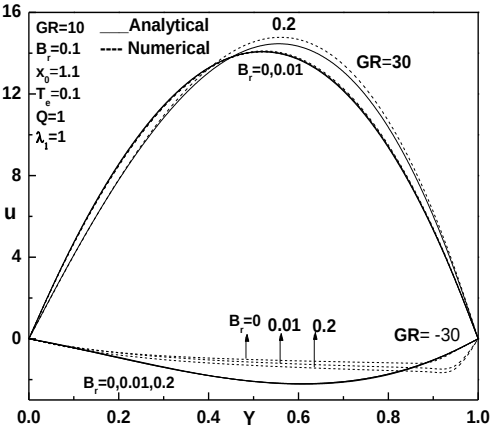

Fig 6.12C: Velocity Profile vs Y different values of B_r (Isoflux-Isoflux Walls)

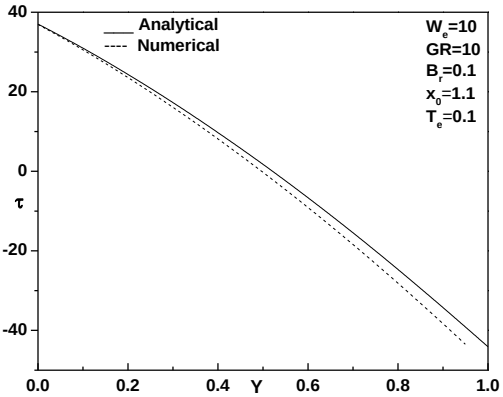

Fig 6.14a: Skin friction vs Y (Isoflux-Isoflux Walls)

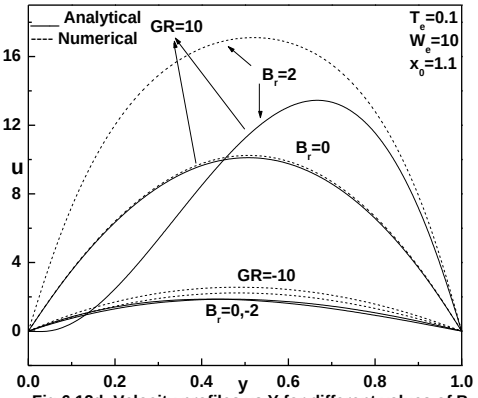

Fig 6.12d: Velocity profiles vs Y for different values of B_r (Isoflux-Isoflux)

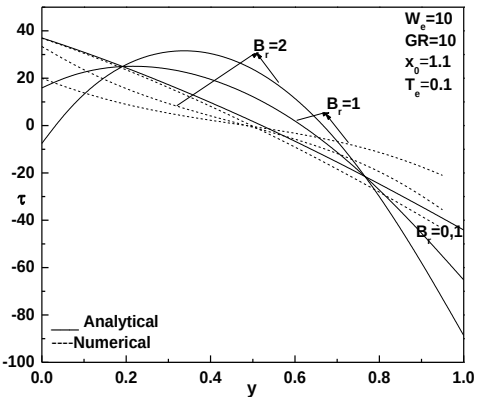

Fig 6.14b: Skin friction vs Y for different values of B_r (Isoflux-Isoflux Walls)

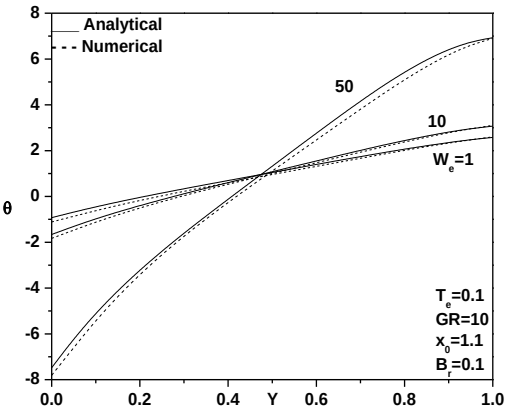

Fig 6.13a:Temperature Profiles vs Y for different values of W_e (Isoflux-Isoflux Walls)

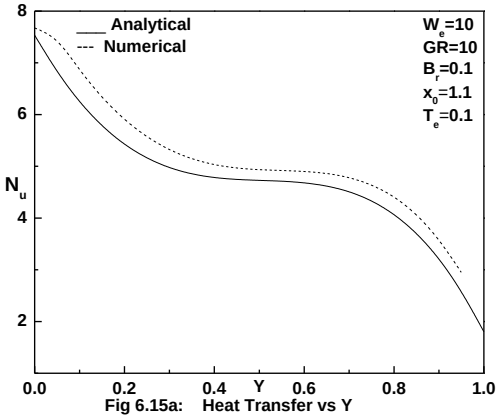

Fig 6.15a: Heat Transfer vs Y (Isoflux-Isoflux Walls)

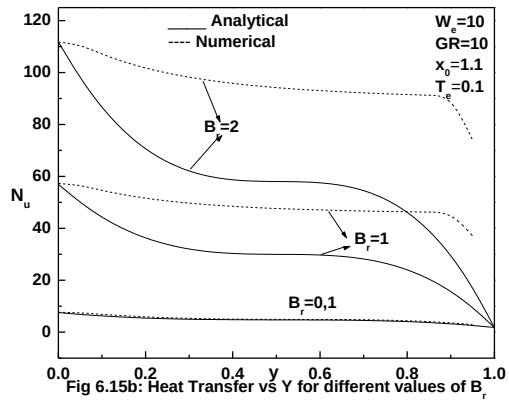

Fig 6.15b: Heat Transfer vs Y for different values of B_r (Isoflux-Isoflux Walls)

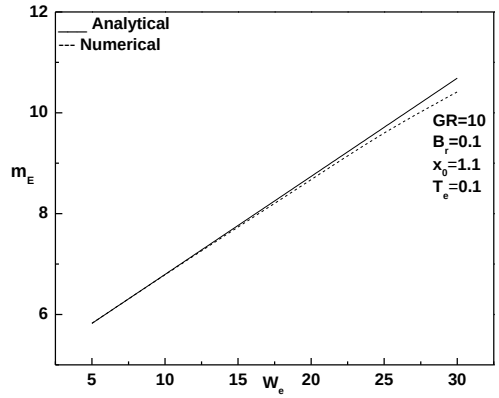

Fig 6.16a: Mass flow rate vs W_e (Isoflux-Isoflux Walls)

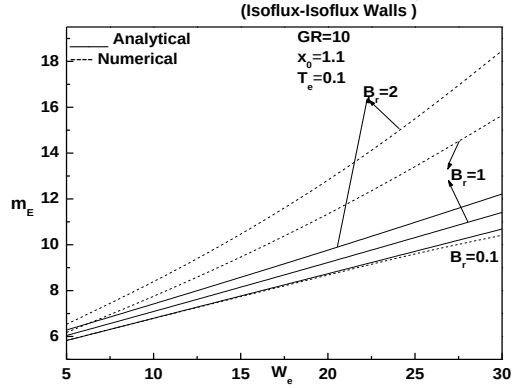

Fig 6.16b: Mass flow rate vs W_e for different values of B_r (Isoflux-Isoflux Walls)

Note: Because of lack of space we omitted the constants.

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