

Research Article

Thermal Stress Analysis of Three Layered Symmetric Laminated Composite Plate

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Abstract

Analytical models in three layered symmetric (0°/90°/0°) laminated composite plate gives shear deformation and transverse normal thermal strains is validated for the thermal stress analysis subjected to gradient thermal profile across the thickness of laminate. Benchmark results are prepared for sinusoidal and parabolic thermal profiles. First Order Shear Deformation Theory is used for calculating various quantities. Numerical results of displacement and stresses are compared with three dimensional elasticity solutions. The paper consists of results which give good agreement with gradient, sinusoidal and parabolic thermal profiles especially for thin plates having aspect ratio more than 20.

Keyword: Composites, Orthotropic, Laminates, Stress analysis, Thermal

1. Introduction

The use of composite materials has increased rapidly during past three decades, particularly in aerospace, underwater and automotive structures. This is largely because many composite materials exhibit high strength-to-weight and stiffness-to-weight ratios, which make them ideally suited for use in weight-sensitive structures. The interlaminar stresses are the main cause of failure when the laminates are subjected to severe thermal loading. This is due to fact that thermal expansion coefficients in the direction of fibers are usually much smaller than in transverse direction resulting high inter laminar stresses at the interfaces. Therefore there is a need to predict inter laminar transverse shear stresses accurately. Various theories and models are reported for the thermal stress analyses of the laminates. In this article First Order Shear Deformation Theory is used for calculating the results.

Most of the researchers assumed linear gradient or constant thermal profiles along the thickness of plates. Laminated composite plate having complexity in material property is assumed in this paper. The primary analysis of three layered (0°/90°/0°), symmetric square laminated composite plate is considered. Material properties are shown in Table 1.

The prime objective of this paper is to suggest the parabolic and sinusoidal thermal profiles across thickness of the plate which is shown in Table 2. This is suggested along with the gradient profile and analytical solutions along with bench mark results in normalization form are prepared.

Table 1 Material properties

Reference	Properties	Normalization
Bhaskar et al. 1996	$\frac{E_L}{ET} = 25,$ $ET = 6.896 GPa,$ $\frac{G_{LT}}{ET} = 0.5,$ $\frac{G_{TT}}{ET} = 0.2,$ $\nu_{12} = \nu_{23} = 0.25,$ $\alpha_L = 0.015 \times 10^{-3} K^{-1},$ $\alpha_T = 0.015 \alpha_L.$	$\bar{w} = \frac{w}{h\alpha_L T_0 S^2},$ $(\bar{u}, \bar{v}) = \frac{(u, v)}{h\alpha_L T_0 S},$ $(\bar{\sigma}_i, \bar{\tau}_{ij}) = \frac{(\sigma_i, \tau_{ij})}{E_T \alpha_L T_0}.$

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Table 2 Proposed thermal load profiles along thickness of plates Boundary Conditions at (a/2, b/2, ± h/2) are

$$-T_0 \leq T \leq T_0$$

Thermal Profile	Equation of profile
TP1	$T = \frac{2z}{h} T_0 \sin(\pi x/a) \sin(\pi y/b)$ (Gradient)
TP2	$T = (2z/h)^3 T_0 \sin(\pi x/a) \sin(\pi y/b)$ (Cubic parabola)
TP3	$T = \sin(\pi z/h) T_0 \sin(\pi x/a) \sin(\pi y/b)$ (Sinusoidal)

Earlier research based on First Order Shear Deformation Theory (FOST) considered in (Reissner, 1945), (Mindlin, 1951) for solutions to include the thermal effects on laminates. About four decades ago (Pagano, 1969) forecasted Exact solution for composite laminated in cylindrical bending and given basic results in material science which are very helpful for today’s research. (Tungikar and Rao, 1994) obtained 3D elasticity solution for temperature distribution and thermal stresses in simply supported rectangular laminates. The actual temperature distribution across the thickness of the laminate is evaluated by solving the Ordinary Differential Equations (ODEs) of heat conduction without internal heat generation. The actual profile also satisfies the interface heat flux continuity. (Kant and Khare, 1994) developed a simple C^oiso-parametric Finite Element (FE) displacement model based on HOST formulations for the analysis of symmetric and asymmetric laminates subjected to thermal gradient. (Bhaskaret al., 1996) [9] developed 3D elasticity solution for laminates under cylindrical and bi-directional bending by assuming linear variation of thermal profile through the thickness of the symmetric laminate. Deficiencies in the FOST (Rower et al., 2001) removed the by incorporating third and fifth order displacement approximations through the plate thickness. 3D elasticity solution can estimate the correct results of the thermally induced quantities like displacements and stresses. (Kant and Swaminathan, 2002) presented simplified formulations through the paper Analytical solutions for static analysis of laminated composite and sandwich plates based on a higher order refined theory and suggested First Order Shear Deformation Theory as a special case with its importance. Interlamiar stresses as well as displacements are computed in laminated composites (Matsunaga, 2004). (Kapuriaet al., 2004) assessed Higher Order Zig-Zag (HZIGT) model along with exact, Zig-Zag (ZIGT) and Third Order Theory (TOT) models for composite laminates subjected various thermal profiles across the thickness. (Kant et al., 2008) developed semi-analytical solution for constant and linear temperature variation through the thickness of a laminate for composites and sandwiches. (Kant and Shiyekar, 2013) developed higher order theory for

composite laminates subjected to thermal gradient using (HOSNT12) model.

2. Formulation

First Order Shear Deformation Theory is used for calculating various results. Mathematical software is used for generating results and results are verified by Matlab.

2.1 First order shear deformation theory (FOST)

In the FOST it is assumed that plane sections originally perpendicular to the middle plane of the plate remain plane but not necessarily normal to the middle plane. This theory was first developed for static bending of plates by Reissner and for free vibration of rectangular plates by Mindlin.

2.1.1 Displacement model

A simply supported single layer orthotropic laminated plate is presented along with complete analytical solution. The geometry of the laminate is such that the side ‘a’ is along ‘x’ axis and side ‘b’ is on ‘y’ axis. The thickness of the laminate is denoted by ‘h’ and is coinciding with ‘z’ axis. The reference mid-plane of the laminate is at h/2 from top or bottom surface of the laminate as shown in the Figure 1. Lamina axes and reference axes coincides with each other. The formulation is assuming fiber direction of the single layered lamina is coinciding with ‘x’ axis of the laminate. Figure also illustrates the mid-plane positive set of displacements along x-y-z axes.

u (x,y,z) , v (x,y,z) , w (x,y,z) are the displacements in x, y, z directions respectively, can be written as:

$$\begin{aligned}
 u &= z\theta_x \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right), \\
 v &= z\theta_y \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right), \\
 w &= w_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)
 \end{aligned}
 \tag{1}$$

The θ_x, θ_y are the rotation of the normal to the middle-plane about y and x-axes respectively. Here for this article, m=n=1 is considered for FOST.

2.1.2 Constitutive Relationship

Stress-strain relationship for orthotropic laminate under linear thermal loading can be written as:

Stress-strain relation for 0^o layer (i.e. top layer and bottom layer),

$$\begin{Bmatrix} \sigma_{x_0} \\ \sigma_{y_0} \\ \tau_{xy_0} \end{Bmatrix} = \begin{bmatrix} Q_{11_0} & Q_{12_0} & 0 \\ Q_{21_0} & Q_{22_0} & 0 \\ 0 & 0 & Q_{33_0} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \tau_{xz_0} \\ \tau_{yz_0} \end{Bmatrix} = \begin{bmatrix} Q_{44_0} & 0 \\ 0 & Q_{55_0} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

Stress-strain relation for 90° layer (i.e. middle layer),

$$\begin{Bmatrix} \sigma_{x_{90}} \\ \sigma_{y_{90}} \\ \tau_{xy_{90}} \end{Bmatrix} = \begin{bmatrix} Q_{11_{90}} & Q_{12_{90}} & 0 \\ Q_{21_{90}} & Q_{22_{90}} & 0 \\ 0 & 0 & Q_{33_{90}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \tau_{xz_{90}} \\ \tau_{yz_{90}} \end{Bmatrix} = \begin{bmatrix} Q_{44_{90}} & 0 \\ 0 & Q_{55_{90}} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{2}$$

Where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz})$ are the stresses and $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the strains with respect to laminate coordinate system (x-y-z).

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z},$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \tag{3}$$

$[Q_{ij}]$ are transformed elastic constants or stiffness matrix and defined as per the following.

$$Q_{11_0} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12_0} = \frac{\nu_{12}E_1}{1 - \nu_{12}\nu_{21}},$$

$$Q_{22_0} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{33_0} = Q_{33_0} = G_{12},$$

$$Q_{44_0} = G_{13}, \quad Q_{55_0} = G_{23}, \quad \frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}.$$

$$Q_{11_{90}} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{12_{90}} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}},$$

$$Q_{21_{90}} = Q_{12_{90}}, \quad Q_{22_{90}} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \tag{4}$$

$$Q_{44_{90}} = G_{23}, \quad Q_{55_{90}} = G_{13}$$

α_x, α_y = Coefficient of thermal expansion in x, y direction, T is temperature profile along thickness direction as shown in Table 1. As stated in eq. (4), the compliance matrix involves engineering properties

namely two extensional moduli (E_1, E_2), two Poisson's ratios (ν_{12}, ν_{21}) and three shear moduli (G_{12}, G_{13}, G_{23}).

3. Discussion

Three layered symmetric (0°/90°/0°) laminated composite square plate made up with material as tabulated in Table 1 is considered for thermal stress analysis. Thermal Profiles 1, 2 and 3 as shown in Table 2 are used for analysis. (Bhaskaret al., 1996) have given the benchmark results for the gradient thermal profile. Fig. 1 A, 1 B and 1 C shows Geometry and co-ordinate system for typical plate and Fig. 2 represents the thermal loading variation of thermal profiles 1, 2 and 3 across the thickness of the laminate.

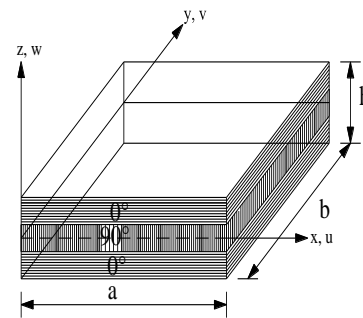


Fig. 1 A Geometry and co-ordinate system for (0°/90°/0°) square plate, (3 D view)

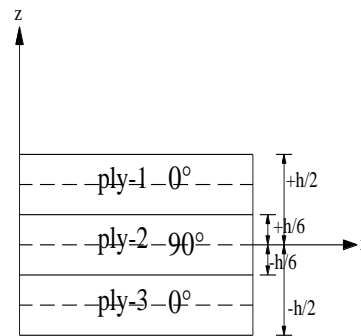


Fig. 1 B Geometry and co-ordinate system for (0°/90°/0°) square plate, (front elevation)

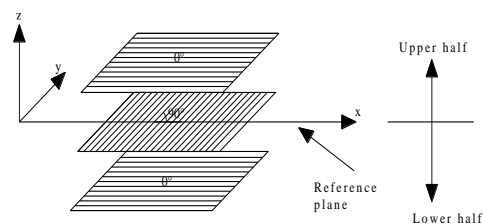


Fig. 1 C Geometry and co-ordinate system for (0°/90°/0°) square plate, (stacking of plates)

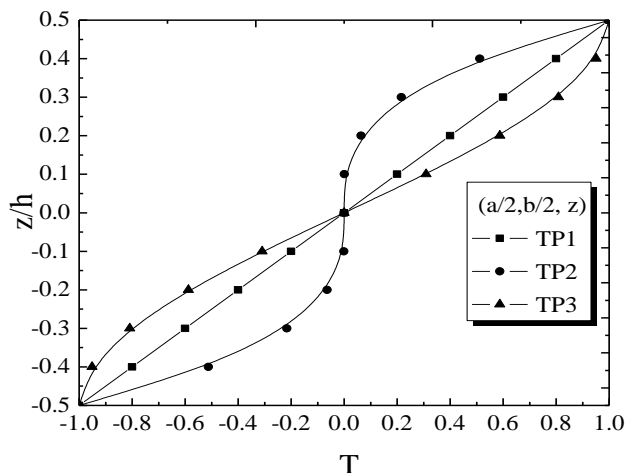


Fig. 2 Thermal loading variation of thermal profiles across the thickness of the laminate

Table 3 Normalized displacements ($\bar{u}, \bar{v}, \bar{w}$)

S=a/h	Thermal Profile	\bar{u} (0,b/2, \bar{T} h/2)	\bar{v} (a/2,0, \bar{T} h/2)	\bar{w} (a/2,b/2, \bar{T} h/2)
20	1	±15.6055 [-3.49]	±19.1072 [-6.06]	10.8901 [-10.14]
	2	±9.3566 [-42.13]	±11.5181 [-43.37]	6.5463 [-45.98]
	3	±18.9779 [17.36]	±23.2 [14.06]	13.2337 [9.19]
	3D ¹	±16.17	±20.34	12.12
50	1	±15.928 [-0.57]	±16.4997 [-1.25]	10.2961 [-1.94]
	2	±9.5557 [-40.35]	±9.9086 [-40.70]	6.1796 [-41.14]
	3	±19.3667 [20.79]	±20.0561 [20.02]	12.5173 [19.21]
	3D ¹	±16.02	±16.71	10.50
100	1	±15.9751 [-0.15]	±16.1185 [-0.31]	10.2092 [-0.49]
	2	±9.5848 [-40.09]	±9.6733 [-40.18]	6.1260 [-40.29]
	3	±19.4236 [19.40]	±19.5964 [21.19]	12.4126 [20.98]
	3D ¹	±16.00	±16.17	10.26

(Bhaskaret al. 1996) =3D¹, Present Thermal Profile = TPⁱ, (i = 1, 2, 3), a/h - Aspect Ratio, [] % Error = (Present Thermal Profile 1or 2 or 3 - Bhaskaret al. 1996) x 100/ Bhaskaret al. 1996).

Table 3 presented the results for variations of normalized transverse displacements \bar{w} and in-plane displacements (\bar{u}, \bar{v}), In plane normal stresses along x and y directions ($\bar{\sigma}_x, \bar{\sigma}_y$) for aspect ratios a/h as 20, 50 and 100. a, b and h are Length, width and the depth

(thickness) of a plate The results are calculated for thermal profiles 1, 2 and 3. The quantities are calculated for z/h= 0.5. Quantities in [] represents % variation with respect to (Bhaskaret al. 1996) results. The quantities are calculated for z/h= 0.5.

Table 4 Normalized stresses ($\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}$)

S=a/h	Thermal Profile	$\bar{\sigma}_x$ (a/2,b/2,h/ 2)	$\bar{\sigma}_y$ (a/2,b/2,h/ 2)	$\bar{\tau}_{xy}$ (0,0,h/2)
20	1	937.565 [-4.52]	-1056.52 [0.525]	-54.574 [-4.84]
	2	439.143 [-55.28]	-1085.37 [3.27]	-32.8185 [-42.77]
	3	1206.55 [22.86]	-1040.97 [-0.95]	-66.3106 [15.62]
	3D ¹	982.00	-1051	-57.35
50	1	960.924 [-0.679]	-1064.49 [0.14]	-50.9817 [-0.83]
	2	453.562 [-53.12]	-1090.29 [2.56]	-30.6011 [-40.47]
	3	1234.72 [27.61]	-1050.57 [-1.17]	-61.9792 [20.55]
	3D ¹	967.5	1063	-51.41
100	1	964.34 [-0.11]	-1065.65 [0.06]	-50.4564 [-0.15]
	2	455.67 [-52.79]	-1091.01 [2.44]	-30.2769 [-40.08]
	3	1238.84 [28.32]	-1051.97 [-1.22]	-61.3459 [21.40]
	3D ¹	965.4	-1065	-50.53

(Bhaskaret al. 1996) =3D¹, Present Thermal Profile = TPⁱ, (i = 1, 2, 3), a/h - Aspect Ratio, [] % Error = (Present Thermal Profile 1or 2 or 3 - Bhaskaret al. 1996) x 100/ Bhaskaret al. 1996)

Table 4 presented the results for variations of normalized stresses ($\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}$), for aspect ratios a/h as 20, 50 and 100. The quantities are calculated for z/h= 0.5.

Table 5 Normalized stresses ($\bar{\tau}_{xz}, \bar{\tau}_{yz}$)

S=a/h	Thermal Profile	$\bar{\tau}_{xz}$ (0,b/2, $\bar{\tau}$ h/6)	$\bar{\tau}_{yz}$ (a/2,0, $\bar{\tau}$ h/6)
20	1	34.6322 [1.91]	-34.9747 [0.61]
	2	21.2418 [-37.48]	-19.2249 [-44.69]
	3	41.8518 [23.16]	-43.4999 [25.14]
	3D ¹	33.98	-34.76
50	1	14.1289 [0.41]	-14.1512 [0.15]
	2	8.66708 [-38.40]	-7.78956 [-44.87]
	3	17.0735 [21.34]	-17.5945 [24.51]
	3D ¹	14.07	-14.13
100	1	7.08462 [0.16]	-7.08742 [0.10]
	2	4.346 [-38.55]	-3.90206 [-44.88]
	3	8.56108 [21.03]	-8.81149 [24.45]
	3D ¹	7.073	-7.080

(Bhaskaret al. 1996) =3D¹, Present Thermal Profile = TP_i (i = 1, 2, 3), a/h - Aspect Ratio, [] % Error = (Present Thermal Profile 1or 2 or 3 - Bhaskaret al. 1996) x 100/ Bhaskaret al. 1996)

Table 5 presented the results for variations of normalized stresses ($\bar{\tau}_{xz}, \bar{\tau}_{yz}$), for aspect ratios a/h as 20, 50 and 100. The quantities are calculated for z/h= 1/6.

Fig.3 shows % error of \bar{u} for thermal profile 1 and 2, which varies from (-3.49,-42.13) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (-0.15,-40.09) respectively for aspect ratio 100. But for thermal profile 3, % error of \bar{u} varies from 17.36 for aspect ratio 20 to 20.79 for aspect ratio 50, while again decreases to 19.40 for aspect ratio 100.

Fig.4 shows % error of \bar{v} for thermal profile 1 and 2, which varies from (-6.06,-43.37) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (-0.31,-40.18) respectively for aspect ratio 100. But for thermal profile 3, % error of \bar{v} varies from 14.06 for aspect ratio 20 increases up to 21.19 for aspect ratio 100.

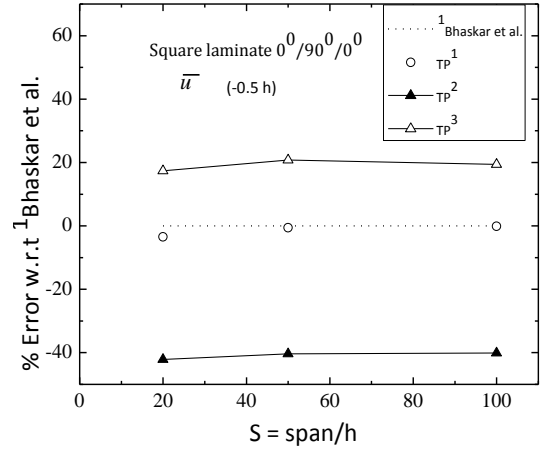


Fig. 3 Variations of % errors of normalized in plane displacement \bar{u} with respect to span to thickness ratio

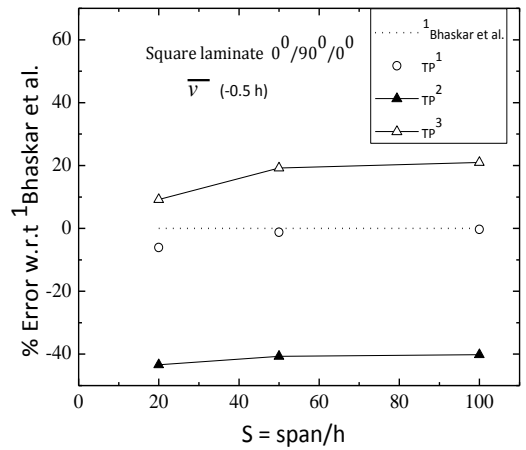


Fig. 4 Variations of % errors of normalized in plane displacement \bar{v} with respect to span to thickness ratio

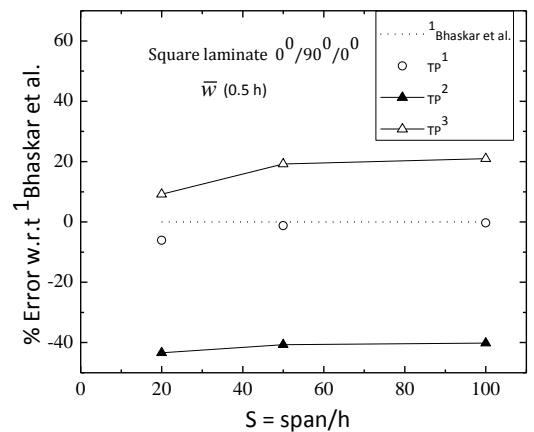


Fig. 5 Variations of % errors of normalized transverse displacement \bar{w} with respect to span to thickness ratio

Fig.5 shows % error of \bar{w} for thermal profile 1 and 2, which varies from (10.14,-45.98) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (-0.49,-40.29) respectively for aspect ratio 100. But for thermal profile 3 % error

of \bar{w} varies from 9.19 for aspect ratio 20 and as a/h ratio increases % error also increases which becomes 20.98 for aspect ratio 100.

increases % error also increases which becomes 21.40 for aspect ratio 100.

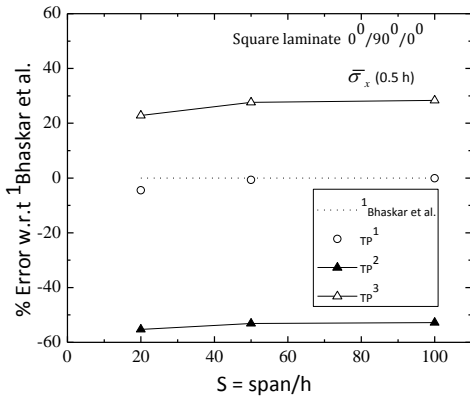


Fig. 6 Variations of % errors of normalized in plane stress $\bar{\sigma}_x$ with respect to span to thickness ratio

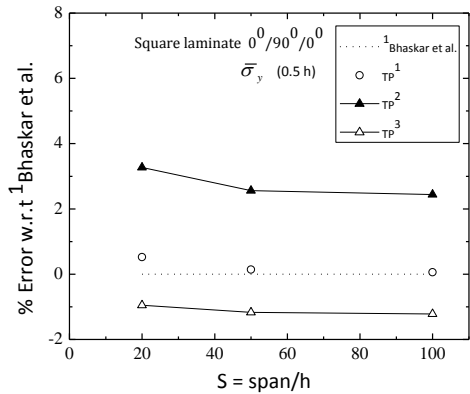


Fig. 7 Variations of % errors of normalized in plane stress $\bar{\sigma}_y$ with respect to span to thickness ratio

Fig. 6 and 7 shows % error of in-plane stresses ($\bar{\sigma}_x, \bar{\sigma}_y$) for thermal profile 1, which varies from (-4.52, 0.52) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (-0.10, 0.06) respectively for aspect ratio 100. But for thermal profile 2, these variations replaced by (-55.28, 3.27) to (-52.79, 2.44) respectively. For thermal profile 3 % error of in-plane stresses ($\bar{\sigma}_x, \bar{\sigma}_y$) , varies from (22.86, -0.95) for aspect ratio 20 and as a/h ratio increases % error increases moderately which becomes (28.32,-1.22) respectively for aspect ratio 100.

Fig. 8 shows % error of in-plane shear stresses $\bar{\tau}_{xy}$ for thermal profile 1 and thermal profile 2, which varies from (-4.84, -42.77) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (-0.15, -40.08) respectively for aspect ratio 100. But for thermal profile 3, % error of $\bar{\tau}_{xy}$ varies from 15.62 for aspect ratio 20 and as a/h ratio

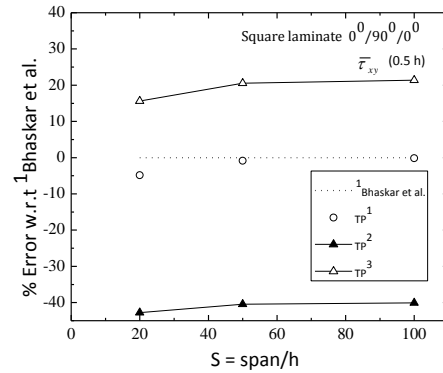


Fig. 8 Variations of % errors of normalized in plane shear stress $\bar{\tau}_{xy}$ with respect to span to thickness ratio.

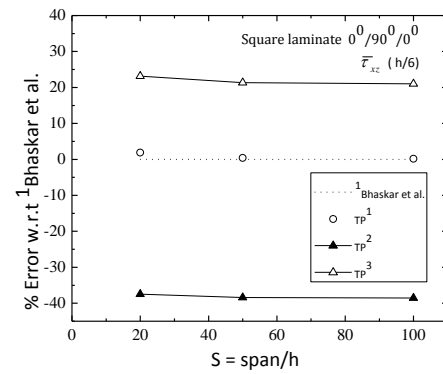


Fig. 9 Variations of % errors of normalized transverse shear stress $\bar{\tau}_{xz}$ with respect to span to thickness ratio

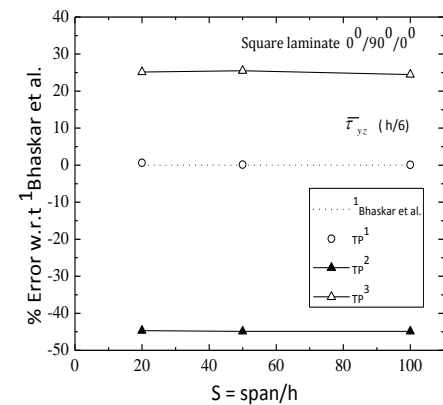


Fig. 10 Variations of % errors of normalized transverse shear stress $\bar{\tau}_{yz}$ with respect to span to thickness ratio

Fig. 9 and 10 shows % error of Transverse stresses ($\bar{\tau}_{xz}, \bar{\tau}_{yz}$), for thermal profile 1, which varies from (1.91, 0.61) for aspect ratio 20 and as a/h ratio increases % error decreases moderately which becomes (0.16, 0.10) respectively for aspect ratio 100.

But for thermal profile 3, these variations replaced by (23.16, 25.14) to (21.03, 24.45) respectively. For thermal profile 2, % error of Transverse stresses ($\bar{\tau}_{xz}, \bar{\tau}_{yz}$), varies from (-37.48,-44.69) for aspect ratio 20 and as a/h ratio increases % error increases moderately which becomes (-38.55,-44.88) respectively for aspect ratio 100.

Conclusions

Three layered symmetric ($0^0/90^0/0^0$) laminated composite square plate under the thermal profiles 1,2 and 3, following concluding remarks are marked.

- 1) Thermal profile 1 gives comparatively good agreement in all the normalized quantities with values given by Bhaskar et al. 1996.
- 2) FOST gives excellent results for thin plates in minimum efforts.
- 3) With some factor of safety and appropriate shear correction factor, designing of thin plates can be recommended instead of complicated computer programs.
- 4) Results are validated for thin plates hence benchmark results are prepared for thermal profile 1, 2 and 3.

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