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Research Article

Application of Constrained Quadratic Adaptive Bacterial Foraging Optimisation Algorithm on a Single Link Inverted Pendulum

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Abstract

This paper presents optimising the control gains of a single link inverted pendulum on cart system using a constrained adaptive bacterial foraging optimisation strategy. The adaptive algorithm is based on the variable chemotactic step for each bacteria in the population to enhance the dispersal of the search and accelerate the convergence of the algorithm. This paper presented the development of simple constraints to bound the search domain within the feasible and stable range of control gains of the system. The system was modelled and simulated in Matlab Simulink software with the linked optimisation algorithm code to simulate the system over multiple iterations. Results showing a great enhancement of the system performance are illustrated and discussed.

Keywords: *optimization, inverted pendulum, bacterial foraging, mobile robot.*

1. Introduction

Evolutionary optimisation algorithms have been widely used recently in research to find the optimal solutions. Evolutionary optimisation was applied into various systems such as mobile robots (almeshal et al, 2013), nonlinear systems (Majhi and Panda, 2007), power distribution systems (Kumar and Jayabarathi, 2012) and data routing in telecommunication networks. Evolutionary algorithms strategies were inspired by nature where swarming effects takes a great role in finding optimum solution in a given domain of solutions. In this paper, adaptive bacterial foraging optimisation algorithm is applied into the classical single inverted pendulum on cart system to find the optimal control gains. The adaptation mechanism has been presented by (Supriyono and Tokhi, 2010) and is based on varying the chemotactic step of the bacterial population in each iteration to yield faster convergence. The chemotactic step can take a variety of functions from linear to highly nonlinear as discussed in (Supriyono and Tokhi, 2012). In this paper, a constrained version of the adaptive bacterial foraging optimisation algorithm is developed for optimising the control gains of an inverted pendulum on cart system.

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2. System description and control strategy

The system considered in this paper is the single link inverted pendulum on cart with two degrees of freedom described in details in [REFERENCE] and represented using state space as:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-(I+ml^2)b}{I(M+m)+Mml^2} & \dfrac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-mlb}{I(M+m)+Mml^2} & \dfrac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{(I+ml^2)}{I(M+m)+Mml^2} \\ 0 \\ \dfrac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

The model will be simulated using decentralised control with two control loops, a cart controller and a tilt angle controller. This allows analysing system behaviour in rejecting disturbances for future work as well as adding more flexibility in testing the optimisation algorithm on multiple variables. Figure 1 represents the block diagram of the control system. The optimisation algorithm will simulate the system on each iteration to acquire the best-cost function that yields the optimal control variables. The optimistaion strategy is discussed in the next section.

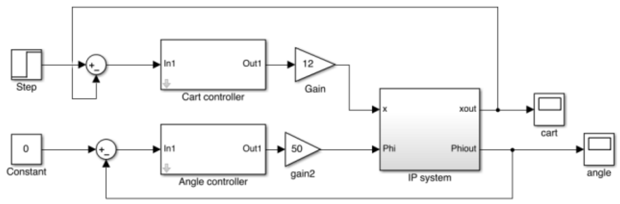

Figure 1 control system block diagram

3. Quadratic adaptive bacterial foraging optimisation

The adaptive-step BFA is a modified version of the original BFA developed by (Passino, 2000) yielding to a faster convergence to the optimum cost function value using a quadratic function as the chemotactic step. The adaptive BFA was developed by (Supriyono, 2012) who has developed a various adaptive BFA algorithm and tested various adaptation mechanisms yielding toward faster convergence to the optimal values. Table 1 provides the symbol notations of the BFA algorithm.

Table 1 BFA optimisation notations

Parameter	Description
P	The dimension of the search space
S	Total number of the bacteria in the population. S must be even.
Nc	Number of chemotactic steps of the bacterium lifetime between reproduction steps
Ns	Number of the swims of the bacterium in the same direction
Nre	Number of reproduction steps
p_{ed}	Probability of bacterium to be eliminated or dispersed
i=1,2,3,…S	Index of bacterium
j=1,2,3,…Nc	Index of chemotaxis
k=1,2,3,…Nre	Index of reproduction steps
l=1,2,3,…Ned	Index of elimination and dispersal events
m_s*=1,2,3,…Ns*	Index of the number of swims
J	The cost function value
C	Step size of the tumble of the bacterium

The following pseudocode describes the original BFA algorithm developed by (Passino, 2000).

The BFA optimisation (Passino, 2000)

1. Elimination and dispersal loop: for $l = 1,2,3,\ldots,N_{ed}$ do $l = l + 1$
2. Reproduction loop: for $k = 1,2,3,\ldots,N_{re}$ do $k = k + 1$
3. Chemotaxis loop: for $j = 1,2,3,\ldots,N_c$ do $j = j + 1$
   a. For $i = 1,2,3,\ldots,S$, take a chemotactic step for bacterium i as follows
   b. Compute the nutrient media (cost function) value $J(i,j,k,l)$ calculate $J(i,j,k,l)$ = $J(i,j,k,l)$ + $J_{cc}(\theta^i(j,k,l),P(j,k,l))$ (i.e., add on the cell-to-cell attractant effect to the nutrient concentration). If there is no swarming effect then $J_{cc}(\theta^i(j,k,l),P(j,k,l))$ =0
   c. Put $J_{last} = J(i,j,k,l)$ to save this value since a better cost via a run may be found.
   d. Tumble: generate a random vector $\Delta(i) \in \Re^r$ with each element $\Delta_{m_r}(i), m_r = 1,2,3,\ldots,r$ a random number on the range [-1,1]
   e. Move: Let $\theta^i(j+1,k,l) = \theta^i(j,k,l) + C(i)\frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}}$ be the result in a step of size C(i) in the direction of the tumble of bacterium i
   f. Compute the nutrient media (cost function) value $J(i,j+1,k,l)$, and calculate $J(i,j+1,k,l)$ = $J(i,j+1,k,l)$ + $J_{cc}(\theta^i(j+1,k,l),P(j+1,k,l))$. If there is no swarming effect then $J_{cc}(\theta^i(j+1,k,l),P(j+1,k,l))$ =0
   g. Swim (note that we use an approximation since we decide behavior of each cell as if the bacteria numbered {1,2,…,*i*} have moved and {*i*+1,*i*+2, *i*+3, …,S} have not; this is much simpler to simulate than simultaneous decisions about swimming and tumbling by all bacteria at the same time):
      i. Let m_s = 0 (counter for swim length)
      ii. While $m_s < N_s$ (if have not climbed down too long)
         1. Count m_s = m_s +1
         2. If $J(i,j+1,k,l)$ < J_{last} (if doing better), then

$J_{last} = J(i, j+1, k, l)$ and calculate

$$\theta^i(j+1,k,l) = \theta^i(j,k,l) + C(i)\frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}}$$

this results in a step of size C(i) in the direction of the tumble for bacterium *i*. Use this $\theta^i(j+1,k,l)$ to compute the new $J(i, j+1, k, l)$ as in step f above.

4. Else let $m = N_s$ (the end of the while statement)
   a. Go to the next bacterium $(i+1)$ if $i \neq S$ (i.e. go to step b above) to process the next bacterium.
5. If $j < N_c$ go to step 3.
6. Reproduction:
   a. For the given k and l, for each $i = 1,2,3,...,S$, let $J^i_{health} = \sum_{j=1}^{Nc+1} J(i,j,k,l)$ be the health of bacterium *i* (a measure of how many nutrients it got over its lifetime and how successful it was at avoiding noxious substances). Sort bacteria and chemotactic parameters C(i) in an ascending order since that higher cost means a lower health.
   b. The S_r bacteria with the highest J_{health} values die and the other S_r bacteria with the best value splot (and the copies that were made are replaced at the same location as their parent)
7. If $k < N_{re}$ go to step 2.
8. Elimination-dispersal: for $i = 1,2,3,...,S$, eliminate and disperse each bacterium which has probability value less than p_{ed}. If one bacterium is eliminated then it is dispersed to random location of nutrient media. This mechanism makes computation simple and keeps the number of bacteria in the population constant

For m=1:S
 If $p_{ed} > rand$ (generate random number for each bacterium and if the generated
 number is smaller than p_{ed} then eliminate/disperse the bacterium)
 Generate new random positions for bacteria
 Else
 Bacteria keep their current position (not dispersed)
 End
end

9. If $l < N_{ed}$ then go to step 1; otherwise end

The quadratic adaptation mechanism presented by (Supriyono, 2012) can be applied by changing the chemotactic step Ci value into a quadratic function as follows:

$$C(i) = \frac{c_{\max}}{1 + \dfrac{a}{b\left((wJ(i))^2 + |wJ(i)|\right)}}$$

where $c_{\max}$ is the maximum allowable chemotactic step value, and tunable positive scaling factors a, b and w. While the cost function is defined as J, the larger the cost function value the smaller the denominator of the function and thus yielding to a larger chemotactic step and vice versa. Therefore, it can be clearly noted that the adaptation mechanism is dependent on the cost function value and may yield to a faster convergence toward the optimum points.

4. Results

The algorithm was applied into a highly nonlinear single link inverted pendulum system with the following simulation parameters:

Table 2 BFA optimisation notations

P	6	Nc	3
S	16	Ns	4
Nre	3	Ned	3
St	S/2	Ped	0.25

Due to the coupled nature of the inverted pendulum system, the controller gains must be limited within the stability range to overcome overshoots and instability. Thus, a constrained optimisation is carried out by having high and low limits for each controller gain. Table 3 presents the control gain limits.

Table 3 Control gain boundaries

	Control gain limits		
	Cart controller		
	Kp	Kd	Ki
High	30	50	3
Low	4	0.1	0.01
	Link controller		
	Kp	Kd	Ki
High	70	50	6
Low	3	0.1	0.08

The convergence plot of the cost function is presented in Figure 2. Optimum values were reached simulating the system and found as:

Table 4 Optimal solutions and cost function

J	0.3588		
Cart controller	Kp	Kd	Ki
	8.91	5.38	1.85
Link controller	Kp	Kd	Ki
	15.4	7.92	0.95

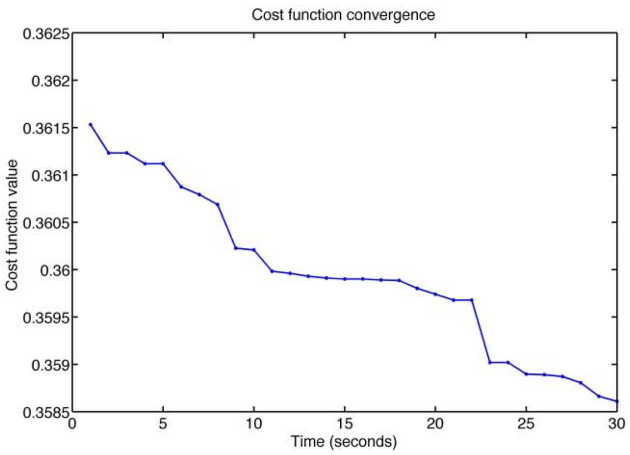

Figure 2 Cost convergence plot

The optimum controller gains were used to simulate the system and compare the optimum response against the initial response with the manually tuned gains. Figures 3 -5 present the comparison of the responses.

The optimal response was found to have a smoother convergence into the set point as well as with less overshoots during the transient phase. Less oscillations can be observed at the pendulum links responses with a stable steady state value.

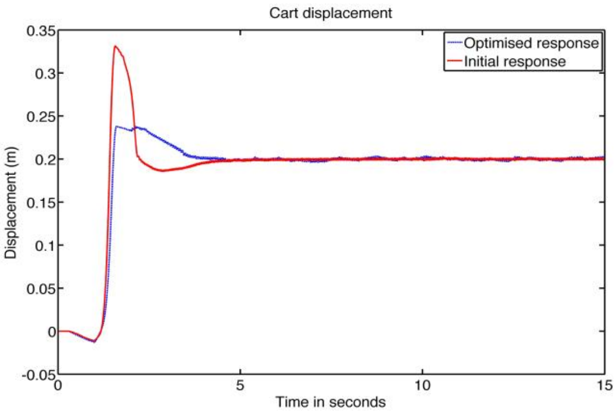

Figure 3 cart response

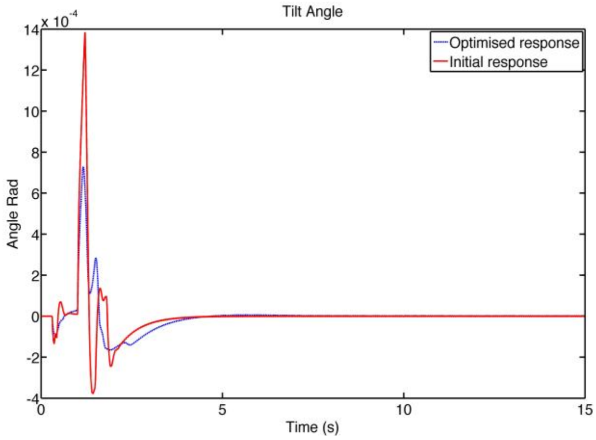

Figure 4 first link response

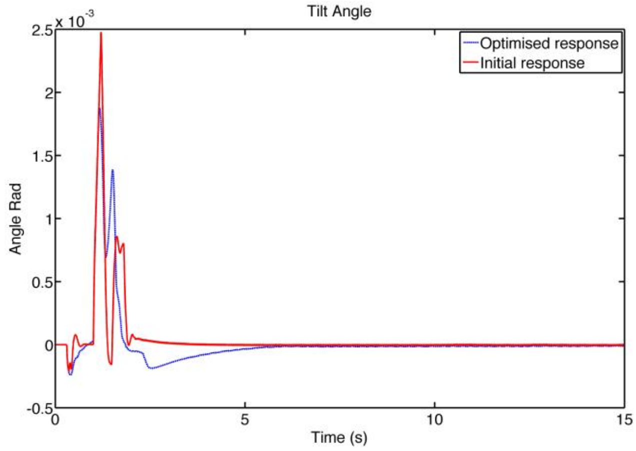

Figure 5 Second link response

Conclusions

This paper has presented a constrained adaptive quadratic bacterial foraging algorithm applied to optimise the control gains of the control loops of the system. The constrained algorithm is proven to work for the highly coupled inverted pendulum system with upper and lower gain limits with a fast convergence time toward the optimal solutions. Results showing a successful stable control within a feasible range of the control gains have been presented and discussed.

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