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Research Article

Designing of Acceptance Sampling Plans through Markov Dependence

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Abstract

Acceptance sampling plays a significant role in quality control. It investigates whether a lot of semi finished or final product satisfies the predetermined quality standards of characteristics of an item of lot in terms of defectives. This paper involves design and development of attribute inspection by Single and Double sampling plans through transition probability matrix in consideration of the threshold value. There may be a Markov dependence in which a given event is dependent to some degree on recent previous events. The main thrust of this paper is to determine optimal threshold value using Markov process which attempts to minimize the expected total cost. The optimal decision is arrived by using absorbing Markov chain for the lot acceptance policy and numerical tables were simulated for the proposed method.

Keywords: *Acceptance sampling, lot acceptance policy, Single sampling plan, Double sampling plan, threshold value, Markov Dependence.*

1. Introduction

Statistical quality control involves statistical methods in monitoring and maintaining of the quality of products and services. In quality control, acceptance sampling is used method to take decision on either to accept or reject the entire lot. Inspection by attributes means count the defectives and inspection by variables is the inspection of quality characteristics by measurements. Sampling plans for count data that work with only one sample of specified size n per lot called Single-sampling plans for attributes and the disposition of the lot is determined from the resulting information. In Double-sampling plans, in order to guard against a wrong decision along with the first sample, a second sample of smaller size is also taken. The idea of the plans is to reduce the average costs by reaching a quick decision. In industry, the manufacturer needs proper construction of threshold value for the number of defectives and average number of units to be inspected in order to avoid financial losses to the manufacturers.

Dodge and Romig (1929) developed an economical method of inspection in Single and Double sampling plans for the purpose to determine the acceptability of discrete lot of product submitted by the producer. Golub (1953) has developed tables for providing values of acceptance number c for different values of specified size n based on the Binomial model for Single sampling plan. Hamaker (1955) studied the efficiency of Double sampling for attributes under the Poisson model. Hald (1967) developed the cost function for inspection and test the efficiency of the sampling plan. Soundararajan (1967) has obtained the acceptance numbers c in the case of Poisson distribution by maximizing the probability of correct decisions. Wilson and Burgess (1971) framed multiple sequential sampling plan as Markov chains and methods for evaluating various properties. Schilling (2009), discussed the number of defects in the sample is distributed according to the Poisson distribution.

Hsiuying Wang and Fugee Tsung (2009) proposed novel procedures for calculating the minimum and average coverage probabilities of Tolerance intervals for Binomial and Poisson distributions. Fallahnezhad and Nasab (2011) introduced a new control policy for the acceptance sampling problem. Fallahnezhad (2012) gave a Markov chain based acceptance sampling through minimum angle method. Mohammad Mirabi and Fallahnezhad (2012) analyse the acceptance sampling on Single and Double stage through Markov chain using Binomial distribution.

This paper involves inspection through Single sampling plan and Double sampling plan by attributes in consideration of the threshold value for the acceptance number in terms of Markovian approach. The upper threshold and the lower threshold optimal value is compared with the number of defective items in each stage of the process. The average number of units inspected and average total cost has been determined.

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2. Transition probability matrix for acceptance sampling

A Markov chain is a special type of stochastic process. Let P = {pij} be the transition probability matrix corresponding to the discrete-time stationary Markov chain with the finite state space S. Transition matrix is the process of moving from one state to another state in one step. It contains all the relevant information regarding the movement of the process among the states.

Procedure

In acceptance sampling, an item in each lot is inspected on a sampling basis and quality of an item is judged.

Assumptions

- All the entries in the matrix are non-negative.
- The sum of the entries in each row is one.

$p_{11}+ p_{12}+ p_{13} +....+ p_{1n} =1$

The three stages involved in sampling are

- Accept the lot.
- Reject the lot.
- Continue the inspection.

Here each stage is considered as states of transition matrix P. The matrix P is an absorbing Markov chain where the states are absorbing (reject or accept the lot) and transient (continue the inspection). This Markovian approach is to determine the optimum threshold value for the number of defectives of the sampling plan.

The transition matrix can be written in the block form given by Wilson and Burgess (1971) as

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix} \qquad (1)$$

Here I= Identity matrix representing the probability p_{ij} of remaining in a state.
0=The null matrix representing the probabilities of not reaching absorbing state.
R=The matrix representing column vector with entry 1-Σqij having probabilities of going from non-absorbing state to absorbing state (from inspection to accept or reject the lot).
Q=The q_{ij} elements representing the square matrix explains the probabilities of going from non-absorbing state to another non-absorbing state.
Expected number of times the inspection continues for the long run m11 can be obtained using the fundamental matrix N,

$$N_{ij} = \frac{1}{(I-Q)} = m_{11} \qquad (2)$$

The Average number of units inspected E(n) is given by

$E(n) = m_{11}$ * sample size per inspection. (3)
The long run absorption probability matrix π can be calculated as $\pi = N_{ij} * R$ (4)

π_{12} = The probability of the lot being accepted.
π_{13} = The probability of the lot being rejected.

Average total cost for the lot acceptance policy for both the sampling plan is expressed as

E(Total cost) = Acceptance cost + Rejection cost + Inspection cost.

Designing of Single sampling plan using Markov chain

The procedure of Single sampling plan is assumed as:

- Take a sample of n items randomly and inspected
- Set the lower threshold value r_1 and upper threshold value r_2
- If the number of defectives observed in the n items are below r_1, then the lot will be accepted.
- If the number of defectives observed in the n items are above r_2, then the lot will be rejected.
- If the number of defectives observed in the n items falls between r_1 and r_2, then the inspection is continued for n more items.

Each steps of the plan are defined as states:

State 1: continue n items inspection.
State 2: the lot is accepted.
State 3: the lot is rejected.

The transition probability matrix of the lot is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Rearranging the matrix gives the fundamental matrix as

$$N_{ij} = \left[\frac{1}{1-p_{11}}\right] = m_{11}$$

The long-run absorption probability π =

$$\begin{bmatrix} \frac{p_{12}}{1-p_{11}} & \frac{p_{13}}{1-p_{11}} \end{bmatrix}$$

Each elements of the π matrix are π_{12} and π_{13} respectively.

The probability of arriving each states are obtained using the cumulative Poisson distribution as $n/N \leq 0.10$, n is large, p is small is given by

$$p(d \le r_1) = p_{12} = \sum_{x=0}^{r_1} \frac{e^{-np}\ np^x}{x!}\ ;\ x = 0,\ldots,r_1$$

$$p(d > r_2) = p_{13} = \sum_{x=r_2+1}^{n} \frac{e^{-np} np^x}{x!} = 1-p_{12} \quad (5)$$

$$p(r_1 < d < r_2) = p_{11} = \sum_{x=0}^{r_2} \frac{e^{-np} np^x}{x!} - \sum_{x=0}^{r_1} \frac{e^{-np}\ np^x}{x!}$$

; $x = r_1+1,\ldots, r_2$

where

p_{11} = The probability of continuing inspection.
p_{12} = The probability of accepting the lot.
p_{13} = The probability of rejecting the lot.

The optimum threshold value r_1 and r_2 can be obtained using desired producer quality level (p_1) and consumer quality level (p_2) associated with producer risk and consumer risk. Ladany (1976) gave the Ladany nomograph of narrow-limit gauging sampling plan with desired p_1=0.02, p_2= 0.08, α and β.

The following conditions must satisfy according to the Golub (1953)

when $p_1 = 0.04$

$$p(r_i) = \sum_{x=0}^{r_i} \frac{e^{-np_1} np_1{}^x}{x!}$$

then $\pi_{12} \ge 1-\alpha$; $\alpha = 0.05$ (a)
when $p_2 = 0.10$

$$p(r_i) = \sum_{x=0}^{r_i} \frac{e^{-np_2} np_2{}^x}{x!}$$

then $\pi_{12} \le \beta$; β=0.10 or $\pi_{13} \ge 0.90$

Average total cost for the lot acceptance policy for Single sampling plan is expressed as

E(Total cost) = $c^*N^*p^*\ \pi_{12} +\ r^*\ \pi_{13} + m_{11}{}^*n^*I$ (6)

Here c = cost of a defective item.

N = lot size
p = incoming quality
n = size of the item inspected.
r = rejection cost for an item.
I = Inspection cost of an item.

Designing of Double sampling plan using Markov chain

The procedure of Double sampling plan is assumed as:

- Take sample of n_1 items randomly and are first inspected
- Set the lower threshold value r_1 and upper threshold value r_2
- If the number of defectives d_1 observed in the n_1 items are below or equal to r_1, then the lot will be accepted.
- If the number of defectives d_1 observed in the n_1 items is above r_2, then the lot will be rejected.
- If the number of defectives d_1 observed in the n_1 items falls between r_1 and r_2, then the second sample n_2 items are inspected.
- Set the lower threshold value r_3 and upper threshold value r_4
- If the number of defectives d_1+d_2 observed are below or equal to r_3, then the lot will be accepted.
- If the numbers of defectives d_1+d_2 observed are above r_4, then the lot will be rejected.
- If the number of defectives d_1 observed falls between r_3 and r_4, then n_1 items are inspected.

Each steps of the plan are defined as states:

State 1: n_1 items inspection applied to the lot.
State 2: n_2 items inspection applied to the lot.
State 3: the lot is accepted.
State 4: the lot is rejected.

The transition probability matrix of the lot is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} \\ p_{21} & 0 & p_{23} & p_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Rearranging the matrix gives the fundamental matrix as

$$N_{ij} = \begin{array}{c} 1 \\ 2 \end{array} \overset{\begin{array}{cc} 1` & \quad 2 \end{array}}{\begin{bmatrix} \dfrac{1}{1-p_{12}p_{21}} & \dfrac{p_{12}}{1-p_{12}p_{21}} \\ \dfrac{p_{21}}{1-p_{12}p_{21}} & \dfrac{1}{1-p_{12}p_{21}} \end{bmatrix}} = m_{ij}$$

The long-run absorption probability π is

$$\begin{array}{c} 1 \\ 2 \end{array} \overset{\begin{array}{cc} 3 & \quad 4 \end{array}}{\begin{bmatrix} \dfrac{1+p_{12}p_{23}}{1-p_{12}p_{21}} & \dfrac{p_{14}+p_{12}p_{24}}{1-p_{12}p_{21}} \\ \dfrac{p_{21}p_{13}+p_{23}}{1-p_{12}p_{21}} & \dfrac{p_{21}p_{14}+p_{24}}{1-p_{12}p_{21}} \end{bmatrix}}$$

Each elements of the π matrix are π_{13} and π_{14} be the probability of accepting the lot at n_1 items inspected and rejecting the lot at n_2 items inspected respectively. The probability of arriving each states are obtained using the cumulative Poisson distribution as

$$p(d \le r_1) = p_{12} = \sum_{x=0}^{r_1} \frac{e^{-np}\ np^x}{x!}\ ;x = 0,\ldots,r_1$$

$$p(d > r_2) = p_{13} = \sum_{x=r_2+1}^{n} \frac{e^{-np} np^x}{x!} = 1-p_{12} \quad (7)$$

$$p(r_1 < d < r_2) = p_{11} = \sum_{x=0}^{r_2} \frac{e^{-np} np^x}{x!} - \sum_{x=0}^{r_1} \frac{e^{-np}\ np^x}{x!}$$

; x = r_1+1,…, r_2

$$p(d \le r_3) = p_{23} = \sum_{x=0}^{r_3} \frac{e^{-np}\ np^x}{x!} \quad ; x = 0,\ldots,r_3$$

$$p(d > r_4) = p_{24} = \sum_{x=r_3+1}^{n} \frac{e^{-np} np^x}{x!} = 1 - p_{23}$$

$$p(r_3 < d < r_4) = p_{21} = \sum_{x=0}^{r_4} \frac{e^{-np} np^x}{x!} - \sum_{x=0}^{r_3} \frac{e^{-np}\ np^x}{x!}$$

; x = r_3+1,…,r_4

where p_{13} = probability that the lot being accepted during the first sample inspected.
p_{14} = probability that the lot being rejected during the first sample inspected.
p_{12} = probability that the second sample is taken during the first sample inspected.
p_{23} = probability that the lot being accepted during the second sample inspected.
p_{24} = probability that the lot being rejected during the first sample inspected.
p_{21} =probability that the first sample is taken during the second sample inspected.
The optimum threshold value r_1, r_2, r_3 and r_4 can be obtained using the similar condition (a)
Average total cost for the lot acceptance policy for

Double sampling plan is expressed as

$$E(\text{Total cost}) = c*N*p*\pi_{13} + r*\pi_{14} + (m_{11}*n_1 + m_{22}*n_2*p_{12})\ I \quad (8)$$

where $\pi_{13} = \frac{p_{13} + p_{12}p_{23}}{1 - p_{12}p_{21}}$, $\pi_{14} = 1 - \pi_{13}$

Here c = cost of a defective item, N = lot size, p = incoming quality, m_{11} = Number of times the lot inspected with n_1 items and m_{22} = Number of times the lot inspected with n_2 items. The study is carried out with the construction of tables for Single sampling plan and Double sampling plans involving distributions of various estimators are tabulated in Table.1, Table.1a, Table 2 and Table.2a

3. Numerical examples on Single sampling plan

The application of the designed method on Single sampling plan is solved numerically in this section. Consider the Single sampling plan with N=1500, c=5, p=0.08, I=2, r=500, n=75.

There are 17 different values of r_1 and r_2 are given among existing alternatives in the Table.1. The bold letters mentioned in the Table.1 in which r_1 and r_2 are feasible values following the condition $\pi_{12} \ge 0.95$ and $\pi_{13} \ge 0.90$.

In the Table.1a, the optimum combination value is r_1=3, r_2=6 when p_1=0.04 and p_2=0.10 with the minimum number of units to be inspected is 138 and their cost of 803.03.

Table 1: The feasible values of r_1, r_2, E(n) and average cost among various values

when p_1=0.04 probability of accepting the lot	when p_2=0.10 probability of rejecting the lot	r_1	r_2	E(n)	Average cost
0.52	0.99	1	4	102	707.21
0.7	0.99	1	5	131	765.42
0.86	0.99	1	6	182	869.14
0.94	0.99	1	7	274	1055.05
0.84	0.97	2	5	122	753.44
0.97	**0.96**	**2**	**7**	**234**	**991.2**
0.99	**0.94**	**2**	**8**	**349**	**1227.41**
1	**0.92**	**2**	**9**	**514**	**1570.63**
0.89	0.93	3	5	106	734.04
0.95	**0.91**	**3**	**6**	**138**	**803.03**
0.98	0.89	3	7	184	905.48
0.99	0.85	3	8	247	1043.22
1	0.78	4	7	139	829.91
1	0.63	4	9	203	983.78
1	0.58	5	8	125	825.12
0.99	0.56	6	7	87	744.26
1	0.47	6	8	99	777.49

Table.1a The optimum values of r_1 and r_2

r_1	r_2	E(n)	Average cost
2	7	234	991.2
2	8	349	1227.41
2	9	514	1570.63
3	6	138	803.03

Numerical examples on Double sampling plan

The application of the designed method on Double sampling plan solved numerically is also given. Let N=1500, c=5, p=0.08, I=2, r=500, n_1=75, n_2 = 65. The bold letters mentioned in the Table2 in which r_1, r_2, r_3 and r_4 are feasible values following the condition $\pi_{13} \ge 0.95$ and $\pi_{14} \ge 0.90$.

In the Table.2a, the optimum combination value is r_1=3, r_2=6, r_3=3, r_4=6 with the minimum number of units to be inspected is 135 and their cost of 803.4 under the approximation of Poisson distribution.

In the Single sampling plan, r_1=3, r_2=6 when p_1=0.04 and p_2=0.10, the α=0.05 and β=0.09 then the minimum number of units inspected and average cost is 138 and their cost of rupees 803.03 respectively and in Double sampling plan r_1=3, r_2=6, r_3=3, and r_4=6, where α=0.04 and β=0.10 then minimum number of units to be inspected is 135 and their cost of rupees 803.4 are more or less same using Poisson approximation Thus reduction in sample size and average Total costs than using binomial distribution on Fallahnezhad (2012). Hence concluded that both sampling plans have better performance using Poisson distribution.

Table 2: The feasible values of r_1, r_2, r_3, r_4, E(n) and average cost among various values

when p_1=0.04 probability of accepting the lot	when p_2=0.10 probability of rejecting the lot	r_1	r_2	r_3	r_4	E(n)	Average cost
0.77	0.99	1	5	1	5	134	772.75
0.91	0.98	1	10	1	5	283	1076.38
0.83	0.99	1	5	1	10	170	846.48
1	**0.93**	**1**	**10**	**1**	**10**	**1333**	**3215.29**
0.83	0.98	1	5	2	5	128	763.59
0.94	0.94	1	10	2	5	247	1015.55
0.88	0.98	1	5	2	10	162	834.21
1	0.81	1	10	2	10	789	2147.92
0.84	0.98	2	5	1	5	125	760.1
0.95	**0.96**	**2**	**10**	**1**	**5**	**264**	**1045.69**
0.87	0.97	2	5	1	10	155	821.35
1	0.86	2	10	1	10	924	2411.21
0.87	0.97	2	5	2	5	121	754.62
0.96	**0.92**	**2**	**10**	**2**	**5**	**233**	**993.46**
0.9	0.96	2	5	2	10	148	812.07
1	0.76	2	10	2	10	630	1836.34
0.97	0.8	4	6	4	6	107	760.66
0.96	**0.9**	**3**	**6**	**3**	**6**	**135**	**803.4**

Table.2a The optimum values of r_1, r_2, r_3, r_4

r_1	r_2	r_3	r_4	E(n)	Average cost
1	10	1	10	1333	3215.29
2	10	1	5	264	1045.69
2	10	2	5	233	993.46
3	**6**	**3**	**6**	**135**	**803.4**

Conclusions

This paper aims to make an attempt in arriving optimum threshold value for the lot acceptance policy using Markovian matrix. Also this methodology initializes the changes happening while minimizing the risks of acceptable quality level and rejectable quality level. The model seems to be efficient in finding the optimal upper threshold and lower threshold value for Single sampling plan and Double sampling plan. The illustrations concluded that both sampling plans have better performance in arriving measures while using Poisson distribution. This benefits both the manufacturer and the customer to arrive at an optimal policy and prevents from financial losses.

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