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Research Article

Design of Spur Gear Considering Contact Stress using Probabilistic Approach

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Abstract

Gears are known for effective transferring of power from one shaft to another by maintaining a definite velocity ratio. Classical method AGMA (American Gear Manufacturing Association) standards are followed to design gears to ascertain the constant transmission of torque and angular speeds of shafts. In reality torque, rotational speed, gear material properties, shaft-alignment and gear geometry are statistical in nature. To account such variability in operating and geometric conditions, the AGMA standards recommend usage of a large number of empirical design factors. Due to subjective nature of those factors, often wrong input design factors lead to over-design or under-design of the gear. To decide the appropriate values of correction factors, it is advisable to account statistical variations in operating and geometric conditions in the form of their mean and standard deviation values. In the present work, a methodology of probabilistic design for durability of spur gear has been presented accounting pitting failure. A case study has been considered to compare the procedure suggested by AGMA standard and the proposed methodology.

Keywords: *Reliability, gear, statistical variation, AGMA.*

1. Introduction

There is a need of mechanical contact between operating gears to transmit the required torque or to operate the shafts at desired speeds. For quiet and vibration less operation of such mechanical contacts, the velocities of the pitch circles of the two gears must be same at all times. An appropriate design methodology is essential to design such gears operating under variable load, speed, and geometric conditions.

Gears are designed to resist bending failure of the teeth and pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength. It was observed by Gautham (2013) that contact factor of safety is more critical than the bending factor of safety. The contact stresses are generally higher. The gears eventually fail to function due to excessive contact stresses and therefore durability of gears is related to contact stresses. Hence in the present paper the surface pitting of gears has been dealt. The pitting is surface fatigue phenomenon caused by stresses exceeding the endurance strength (corrected endurance limit) of the gear material.

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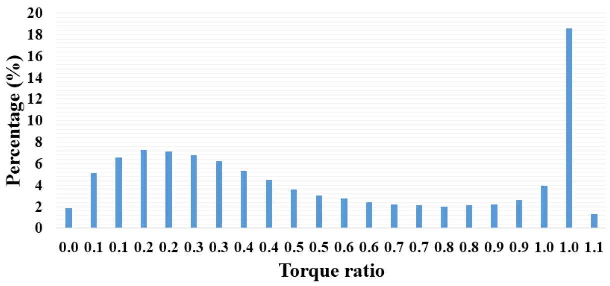
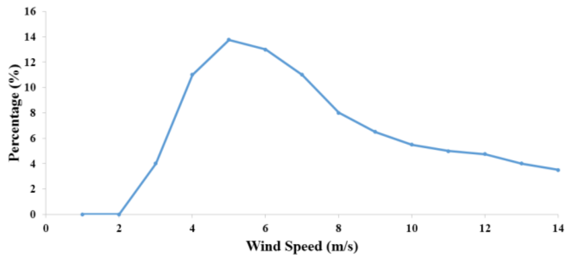
The pitting failure is related to the contact stress. Various formulae for the pitting stress are provided in standards of AGMA (American Gear Manufacturing Association). The AGMA based contact stress equation incorporates a number of correction factors such as overall load factor, dynamic factor, load distribution factor, surface condition factor etc. The values/expression of these factors are based on large numbers of tests and many years of experience and do not consider the actual uncertainties in the input data, make these factors less robust. Often the values of design factors, decided by designers, are very subjective.

Even though AGMA has provided design guidelines, proper gear design is a challenging task and it becomes more challenging in case of fluctuating operating conditions, variable material properties and statistical geometric dimensions. Variation in operating conditions in one of typical gearbox used in wind-power-generator is presented in figure 1. There is a need of statistical design approach to deal with such variations in operating conditions. It is interesting to note that statistical variation in operating conditions is very common in most of mechanical components such as bearings [Hirani, 2009, Hirani *et al*, 2000, Hirani *et al*, 1999, Hirani *et al*, 1998, Muzakkir *et al*, 2011, Hirani, 2005, Hirani *et al*, 2001, Muzakkir *et al*, 2013, Hirani 2004, Muzakkir *et al*, 2015, Hirani, Verma, 2009, Hirani, Suh, 2005, Hirani *et al*, 2001, Rao *et al*, 2000, Hirani *et* *al*, 2000, Hirani *et al*, 2002, Burla *et al*, 2004, Hirani, Samanta, 2007, Lijesh, Hirani, 2015, Lijesh, Hirani, 2014, Shankar *et al*, 2006, Lijesh, Hirani, 2015, Muzakkir *et al*, 2014, Lijesh, Hirani, 2015], seals [Hirani and Goilkar, 2011, Goilkar and Hirani, 2010, Hirani and Goilkar, 2009, Goilkar and Hirani, 2009, Goilkar and Hirani, 2009], brakes [Sarkar and Hirani, 2015, Sarkar and Hirani, 2015, Sarkar and Hirani, 2013, Sarkar and Hirani, 2013, Sarkar and Hirani, 2013, Sukhwani *et al*, 2009, Sukhwan and Hirani, 2008, Sukhwani *et al*, 2008, Sukhwani *et al*, 2008, Hirani and Manjunath, 2007, Sukhwani *et al*, 2007], gears [Shah and Hirani, 2014, Hirani, 2009]. The proposed probabilistic design approach can be applied to any of aforementioned machine component.



(a) Torque ratio variation



(b) Wind Speed variation

Fig. 1 Torque and wind speed variation of wind turbine (Issa *et al* 2014)

There is need to develop a design methodology to incorporate the statistical variation in load, speed, material properties, and gear dimensions. Implementation of such methodology will eliminate the need of subjective-correction factors advised by AGMA. In the present work, a methodology based on mean and standard deviation values of design factors has been presented. To exemplify the proposed design procedure, a case-study considering is presented. Each step has been described. The obtained results are matched with results obtained from AGMA procedure.

2. Gear Design

The designer involved with gears is expected to follow the pertinent standards of the AGMA (classical) approach using involute gearing profile. AGMA standard 2001-B88 assumes following related to mesh geometry (Norton, 2001):

i. Contact ratio is between 1 and 2.
ii. There is no interference between the tips and root fillets of matting teeth.
iii. No undercutting.
iv. No tooth is pointed
v. There is nonzero backlash.
vi. The root fillets are standard, assumed smooth and produced by a generating process.
vii. The friction forces are neglected.

In the present work, a summary of AGMA approach has been presented. To design gears, the proposed probabilistic approach has been detailed. A case study to compare the AGMA and the proposed design approaches presented.

(i) Classical approach (Budynas and Nisbett, 2014)

The contact stress given by AGMA approach is given by equation (1).

$$\sigma_c = C_p \sqrt{\frac{W_t K_v K_o K_s K_H C_f}{FdI}} \qquad (1)$$

Here; W_t is tangential load (N), F is face-width, m is gear-module and K_v is dynamic factor. K_v is represented as:

$$K_v = \frac{3.05+V}{3.05} \qquad \text{(Cast iron, cast profile)} \qquad 2(a)$$

$$K_v = \frac{6.1+V}{6.1} \qquad \text{(Cut or milled profile)} \qquad 2(b)$$

$$K_v = \frac{3.56+\sqrt{V}}{3.56} \qquad \text{(Hopped or shaped profile)} \qquad 2(c)$$

$$K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}} \qquad \text{(Shaved or ground profile)} \qquad 2(d)$$

K_o is the overload factor as listed in Table 1.

Table 1 Table of overload Factor

Power Source	Uniform	Moderate shock	Heavy Shock
Uniform	1	1.25	1.75
Light Shock	1.25	1.5	2
Medium Shock	1.5	1.75	2.25

K_s, size factor, as a function of gear-module, face width and geometry is given as:

$$K_s = 1.192\left(Fm\sqrt{Y}\right)^{0.0535} \qquad (3)$$

K_H, load distribution factor, is very complicated. It accounts non-uniformity in distribution of load across the line of contact and expressed by equation (4).

$$K_H = 1 + C_{mc}\left(C_{pf} C_{pm} + C_{ma} C_e\right) \qquad (4)$$

Where;

$$C_{mc} = \begin{cases} 1 & \text{For uncrownedteeth} \\ 0.8 & \text{For crownedteeth} \end{cases}$$

$$C_{pf} = \begin{cases} \frac{F}{10D} - 0.025 & F < 25mm \\ \frac{F}{10D} - 0.0375 + F4.92\times10^{-4} & 25mm < F < 425mm \\ \frac{F}{10D} - 0.1109 + F8.15\times10^{-4} - F^2 3.53\times10^{-4} & 425mm < F < 1000mm \end{cases}$$

If F/(10D)<0.05, F/(10d)=0.05 is used

$$C_{pm} = \begin{cases} 1 & \text{For straddle mountedpinion with } S_1/S < 0.175 \\ 1.1 & \text{For straddle mountedpinion with } S_1/S \ge 0.175 \end{cases}$$

S is the center distance between two bearings and S_1 is the distance between the center line of the gear face and mid-point of shaft-system as shown in figure 1.

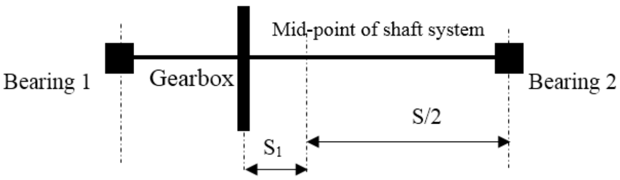


Fig.1 Definition of distance S and S_1 used in evaluation C_{pm}

$$C_{ma}=A+BF+CF^2$$

The value of A, B, and C is given in table 2.

Table 2 Empirical Constants A, B, and C to find C_{ma}

Condition	A	B	C
Open gearing	0.247	6.57x10^{-4}	-3.01x10^{-6}
Commercial, enclosed units	0.127	6.22x10^{-4}	-3.66x10^{-6}
Precision, enclosed units	0.0675	5.03x10^{-4}	-3.65x10^{-6}
Extra precision enclosed gear units	0.00360	4.015x10^{-4}	-3.24x10^{-6}

$$C_e = \begin{cases} 0.8 & \text{For gearing adjusted at assembly or compatability is improved by lapping or both} \\ 1 & \text{For all other conditions.} \end{cases}$$

The elastic coefficient (C_p) is given by,

$$C_p = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_p}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right]}} \quad (5)$$

Where; E_P and E_g are the moduli of elasticity and ν_P and ν_g are the Poisson's ratios for pinion and gear respectively.

The surface geometry factor (I) takes into account the radii of curvature of the gear teeth and the pressure angle. As per AGMA I is given by equation (6).

$$I = \frac{Cos\phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right)D} \quad (6)$$

Where;

$$\rho_p = \sqrt{\left(\frac{D}{2}+(1+x_p)m\right)^2 - \left(\frac{D}{2}cos\,\phi\right)^2} - m\pi\cos\phi \quad 7(a)$$

$$\rho_g = C\,sin\phi \mp \rho_p \quad 7(b)$$

Where D is the pitch diameter of the pinion, ϕ is the pressure angle, and C is the center distance between pinion and gear. The negative sign in equation 7(b) is for the external gear-set, while positive sign is for the internal gear-set. C_f is the surface condition factor.

To avoid pitting failure, equation (8) must be satisfied

$$\sigma_c \le \frac{S_c Z_N Z_w}{K_T K_R S_{Fc}} \quad (8)$$

Where; S_C is the allowable contact stress based on hardness value, Z_N is the stress cycle factor, Z_w are the hardness ratio factor for pitting resistance, K_T is the temperature factor, K_R is the reliability factor and S_{Fc} is the factor of safety.

The value of allowable contact stress is given by equation 9(a).

$$S_c = \begin{cases} 2.22H_B + 200MPa & \text{For Grade 1} \\ 2.41H_B + 237MPa & \text{For Grade 2} \end{cases} \quad 9(a)$$

The stress cycle factor (Z_N) is given by the graph provided in figures 14-15 of book of Richard, Budynas (2014). The reliability factor (K_R) is estimated based on the required reliability (R) and the value is estimated from equation 9(b).

$$K_R = \begin{cases} 0.658 - 0.0759\,ln(1-R) & 0.5 < R < 0.99 \\ 0.50 - 0.109\,ln(1-R) & 0.99 < R < 0.9999 \end{cases} \quad 9(b)$$

For oil or gear blank temperature up to 120^{0}C, K_T is considered to be unity. For higher temperature the factor should be greater than unity. Z_w is calculated by equation 9(c) or 9(d). For through-hardened pinions running against through hardened gears.

$$Z_w = 1 + A(m_G - 1) \quad 9(c)$$

Where m_G is the gear ratio, A is the calculated for following condition.

$$\begin{cases} if\ \frac{HB_p}{HB_g} < 1.2 \quad then \quad A = 0 \\ if\ 1.2 < \frac{HB_p}{HB_g} < 1.7 \quad then \quad A = 0\ .00898\frac{HB_p}{HB_g} - 0.00829 \\ if\ \frac{HB_p}{HB_g} > 1.7 \quad then \quad A = 0.00698 \end{cases}$$

For surface-hardened pinions (>48HRC) run against hardened gears, Z_w is found from:

$$Z_w = 1 + B(450 - H_g) \quad 9(d)$$

$B = 0.00075e^{-0.052R_q}$

Where, R_q is the rms surface roughness of the pinion teeth in μm.

(ii) Proposed Probabilistic Design Approach

To overcome the problems related to classical approach, probabilistic approach incorporating the statistical variation in various design factors has been used. In this approach, it is assumed that the standard deviation of each variable follows the Gaussian distribution, such as:

$$f(Q) = \frac{1}{\hat{\sigma}_Q\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{Q-\mu_Q}{\hat{\sigma}_Q}\right)^2\right] \quad (10)$$

Here μ_Q is mean and $\hat{\sigma}_Q$ is standard deviation of the variable concerned. To normalize the Eq. (10), concept of 'normal variable' (Equation 11), having a mean of zero and a standard deviation of unity, can be used.

$$Z = \frac{Q-\mu_Q}{\hat{\sigma}_Q} \quad (11)$$

The probability distribution curves of stress (σ) and strength (S_y) have been shown in figure 4. The mechanical component will not fail if the strength is greater than the stress. In other words, the margin of safety for any value of stress σ and strength S_y is defined as $Q=S_y-\sigma$. Any chance of $Q < 0$, results in the probability of component failure. The reliability (R) that a part will perform without failure is the area of the margin of safety distribution for $Q > 0$. The interference is the area (1−R) where part fails.

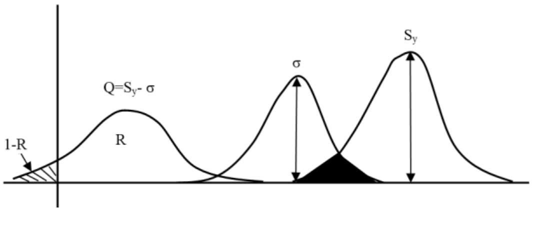


Fig. 4 Stress and strength distributions

To find the chance that Q>0, the value of 'Z' variable (equation 11) is obtained for Q=0.

$$Z = \frac{Q-\mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} \quad (12)$$

Where $\mu_Q = \mu_S - \mu_\sigma$ and $\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\sigma^2}$.

From the value of Z, the reliability of the component is estimated using the table given in A-10 of book authored by Budynas, Nisbett (2014).

In the proposed approach, the mean and standard deviation of the induced stress is calculated. To find the mean value of stress, mean values of all design variables is considered. To account standard deviation of stress, stress equation is differentiated w.r.t. each individual independent variable. If the design variable is dependent then it is substituted as function of independent variables. To understand the procedure, consider the equation (1).

$$\sigma_c = C_p\sqrt{\frac{W_t K_v K_o K_s K_H C_f}{FdI}} \quad (1)$$

Here K_v is function of velocity, which is function of pitch diameter and rotational speed, therefore suitable expression of K_v from Eq. (2) shall be substituted in Eq. (1). Similarly I is a function of pressure angle ϕ, as expressed in Eq. (6). Interesting the running pressure angle is function of operating distance between centers of gears. During assembly there is a possibility of variation in center distance, due to which nominal pitch circle diameter and pressure angle (ϕ) vary. As base pitch remains constant, pressure angle can be expressed as:

$$\phi = cos^{-1}\left(\frac{r_{p\,\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{r_p}\right) \quad 13(a)$$

'nom' is the suffix for 'nominal'. Pitch circle radius (r_p) can be replaced in terms of pitch circle diameter (D), such as:

$$\phi = cos^{-1}\left(\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{D}\right) \quad 13(b)$$

D_{nom} is given as module*number_of_teeth

$$\phi = cos^{-1}\left(\frac{mZ_p\,cos\,\phi_{\mathrm{nom}}}{D}\right) \quad 13(c)$$

This expression of pressure angle can be substituted in expression of 'I' as module (m), number of teeth (Z_p) and nominal pressure angle (ϕ_{nom}) remain constant. Once these parameters are selected, their values remain constant.

Similarly, substituting values in Eq. (7)

$$\rho_p = \sqrt{\left(\frac{D}{2}+m\right)^2 - \left(\frac{D}{2}\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{D}\right)^2} - m\pi\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{D}$$

$$\rho_p = \sqrt{\left(\frac{D}{2}+m\right)^2 - \left(\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{2}\right)^2} - m\pi\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{D} \quad 13(d)$$

Centre distance can be represented in diameter of pinion (D) and gear ratio (m_g) as given in equation 7(d)

$$\rho_g = C\,sin\phi - \rho_p$$

$$\rho_g = \frac{D(1+m_g)}{2} sin\left(cos^{-1}\left(\frac{D_{nom}\,cos\,\phi_{\mathrm{nom}}}{D}\right)\right) - \sqrt{\left(\frac{D}{2}+m\right)^2 - \left(\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{2}\right)^2} + m\pi\frac{D_{\mathrm{nom}}\{cos\,\phi_{\mathrm{nom}}\}}{D} \quad 13(e)$$

Substituting equations 13(d) and 13(e) in equation (6) for external gear we get;

$$I = \frac{\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{D}}{\left(\frac{\rho_g+\rho_p}{\rho_p\rho_g}\right)D} \Rightarrow \frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{\left(\frac{D^3(1+m_g)\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{D}}{\rho_p\rho_g}\right)} \Rightarrow \frac{\rho_p\rho_g}{D^2(1+m_g)}$$

$$I = \frac{\left\{\begin{array}{l}\left[\frac{D(1+m_g)}{2}\sin\left(\cos^{-1}\left(\frac{mZ_p\cos\phi_{\text{nom}}}{D}\right)\right)\times \left[\left(\sqrt{\left(\frac{D}{2}+m\right)^2-\left(\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{2}\right)^2}-m\pi\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{D}\right)\right]\right] \\ \pm\left[\sqrt{\left(\frac{D}{2}+m\right)^2-\left(\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{2}\right)^2}-m\pi\frac{D_{\text{nom}}\{\cos\phi_{\text{nom}}\}}{D}\right]^2\end{array}\right\}}{D^2(1+m_g)} \quad 13(f)$$

The stress equation given in equation (1) is can be rewritten as a function of K_v, W_t, F, K_o, K_s, K_H, C_f, D, I and C_p as given in equation (14).

$$\sigma_c = f(K_v, W_t, F, K_o, K_s, K_H, C_f, D, I, C_p) \quad (14)$$

The justification of considering the variables as dependent and independent is given below:

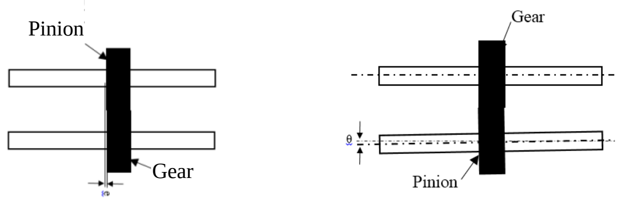
(i)The variable load (W_t) is dependent on the applied torque (T) and pitch diameter (D) (W_t=2T/D). Therefore *f(W_t)* is replaced by *f(T,D).*

(ii)K_v is function pitch line velocity (V) (given equation (2)) and V is function of angular speed (N) and pitch diameter (D) (V=πDN/60).

(iii)In the present approach variable in load is accounted by considering variations in torque and speed; therefore there is no need to consider the over load factor (K_o).

(iv)The load distribution factor (K_H) is the due to the improper engagement of gear and pinion as shown in figure 5. Due to the assembly error in engaging the gear and pinion, as well in the mounting of bearings, the faces of gear and pinion shift and as a result effective face width in contact decreases.

It is interesting to note that gear effective width depends on the alignment and rigidity of the support bearings. Figure 5(a) depicts an axial offset between the gears, which results in reduction in contacting face width. Similarly an angular offset is shown in figure 5(b). These offsets arise due to the clearance in the bearings and coupling (i.e. Jaw coupling) between driving and driven shafts. Therefore to incorporate the design factor for load distribution, the full assembly has to be considered. This problem can be overwhelmed by including the statistical variation considering the full system. However to get the accurate results, intensive study has to be performed which is out of scope of the present work. In statistical approach, this can be accounted by consider the minimum value of face width. To understand this let us consider mean value of width as μ_b and standard deviation as σ_b. In statistical approach, the minimum value of face width will be μ_b-3σ_b. By considering this, there is no need to account K_H and standard deviation in face width. From equation (4) $K_H=f(F,D)$.



(a) Axial offset (b) Angular Offset

Fig. 5 axial and angular offset during gear and pinion engagement

(v)In statistical approach variation in material properties is accounted. In addition, the minimum possible value of face width is considered. Therefore, there is no need to account size factor K_s.

(vi)There is very rare chance of designing thin rim of gear. Generally t_r is far greater than the tooth thickness (m_B>1.2). The basic reason for stating m_B>1.2 is that the diameter of shaft is generally much smaller than the pitch diameter of the gear. Therefore, there is no need to account K_B.

(vii)In equation 7(e), the value of m_g, Z_p and P_c are constant for any type of gear system, hence *'I'* is the function of diameter (D). As the Young's modulus and poison's ratio will be remaining constant the value of C_p shall be considered as constant.

From the above discussion, equation (14) is reduced to equation (15).

$$\sigma_c = f(T, F, D, N) \quad (15)$$

$$\sigma_c = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_P}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right]}}\sqrt{\frac{Tf(D,N)f(F,D)}{FD^2 f(D)}} \quad (16)$$

The standard deviation of equation (16) is estimated by equation (17)

$$\sigma_c = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial\sigma_c}{\partial x_i}\right)^2 \partial x_i^2} \quad (17)$$

$$\sigma_c = \sqrt{\left(\frac{\partial\sigma_c}{\partial D}\right)^2\sigma_D^2 + \left(\frac{\partial\sigma_c}{\partial T}\right)^2\sigma_T^2 + \left(\frac{\partial\sigma_c}{\partial F}\right)^2\sigma_F^2 + +\left(\frac{\partial\sigma_c}{\partial N}\right)^2\sigma_N^2} \quad (18)$$

The mean value (μ_σ) of stress is obtained by substituting the mean value of each variable given in equation (1).

To examine the variation of solutions attained using with and without statistical variation of AGMA equation (1), a case study is considered and comparison of results is presented.

Case Study: Determine durability of a high quality (shaved and grounded) pinion of spur gear system to transmit a torque of 1130Nm @ 4500 rpm with speed reduction of 3.5. Assume the pressure angle = 20°, number of teeth (Z_p)=29 module(m)=6mm. The corrected endurance strength of gear material varies in the range of 300 MPa to 400 MPa. Standard deviations in torque is σ_T= 300 Nm, in speed 100 rpm and in pitch diameter 0.5mm. Due to crowning and manufacturing errors, the face width varies from 50mm to 52mm. Due to possibilities of angular and parallel offset, effective face width may varies in the range of 40mm to 52mm. Considered the gear is made of steel material (210GPa).

Solution: To determine the durability of spur gear, one of the two approaches (i) AGMA classical approach and (ii) Proposed probabilistic approach can be used. In the present case, both the approaches have been used and comparison among the obtained results has been presented.

(i) Classical Approach

The values of correction factors:

$$K_v = \sqrt{1+0.0412\sqrt{DN}}$$

$$K_s = 1.192\left(F_{min} m\sqrt{Y}\right)^{0.0535}$$

$$\Rightarrow 1.192\left(0.05\times 0.006\times\sqrt{0.357}\right)^{0.0535} \Rightarrow 0.7513<1 \Rightarrow K_s = 1$$

$$K_H = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \Rightarrow 1.135$$

The stress calculated using the AGMA equation given in equation (1) is estimated as follows:

$$\sigma_c = C_p\sqrt{W_t K_v K_o K_s K_H \frac{C_f}{FDI}}$$

$$\sigma_c = 1.91\times 10^5\sqrt{\frac{1130}{0.174}\frac{\sqrt{1+0.0412\sqrt{DN}}\times 1.25\times 1\times 1.13\times 1}{0.051\times 0.174\times 0.1201}}$$

$$\Rightarrow \sigma_c = 678 MPa$$

$$FOS = \frac{(800+600)}{2\sigma_Q} = 1.03$$

FOS of safety is 1.03.

(ii) Probabilistic Approach

As per provided information, the dynamic factor for shaved and grounded pinion is given as:

$$K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}} \tag{19}$$

The pitch line velocity (V) is a function of pitch circle diameter of pinion (D) and speed (N). On incorporating these values, equation (16) is rewritten as:

$$K_v = \sqrt{\frac{5.56+\sqrt{\frac{\pi DN}{60}}}{5.56}} \Rightarrow \sqrt{1+0.0412\sqrt{DN}}$$

$$C_p = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_P}\right)+\left(\frac{1-\nu_g^2}{E_g}\right)\right]}} \Rightarrow 1.91\times 10^5 Pa^{0.5}$$

$$I = \frac{\left\{\begin{aligned}&\left[(1+m_g)D_{nom}\left((\cos\phi_{nom})\sqrt{\left(\frac{D}{2}+m\right)^2-\left(\frac{D_{nom}\{\cos\phi_{nom}\}}{2}\right)^2}-m\pi\frac{D_{nom}\{\cos\phi_{nom}\}}{D}\right)\right]\\ &\pm\left[\sqrt{\left(\frac{D}{2}+m\right)^2-\left(\frac{D_{nom}\{\cos\phi_{nom}\}}{2}\right)^2}-m\pi\frac{D_{nom}\{\cos\phi_{nom}\}}{D}\right]^2\end{aligned}\right\}}{D^2(1+m_g)}$$

$$I = \frac{\left\{\left[0.654\times\left(\sqrt{\left(\frac{D}{2}+0.006\right)^2-0.0067}-\frac{0.0031}{D}\right)\right]+\left[\sqrt{\left(\frac{D}{2}+0.006\right)^2-(0.0067)^2}-\frac{0.0031}{D}\right]^2\right\}}{4D^2} \tag{20}$$

Substituting the above estimated values in equation (21) we get:

$$\sigma_c = 1.91\times 10^5\sqrt{\frac{T\sqrt{1+0.0412\sqrt{DN}}}{FD^2 I}} \tag{21}$$

Partial differentiation equation (21) w.r.t. D, N, T andF is difficult. Therefore in the present the differentiation is performed using MATLAB software.

$$\frac{\partial\sigma_s}{\partial T} = 1.38\times 10^5, \frac{\partial\sigma_s}{\partial N} = 4.65\times 10^3, \frac{\partial\sigma_s}{\partial D} = -1.38\times 10^9, \frac{\partial\sigma_s}{\partial F} = -3.127\times 10^9$$

Given σ_T=300 Nm, σ_N= 100rpm, σ_F= 2mm, and σ_D= 0.5mm. Substituting the values obtained and given values in equation (15), the estimated value of σ_s is 46.97MPa. Now to calculate the value of normalized factor 'Z', μ_{sy}, σ_{sy}, μ_Q, σ_Q is calculated as follows

σ_{sy}=(400-300)/6=16.67MPa,

μ_{sy}=(400+300)/2=350MPa.

$$\sigma_c = C_p\sqrt{\frac{W_t K_v}{FDI}} \Rightarrow 1.91\times 10^5\sqrt{\frac{1130}{0.174}\frac{\sqrt{1+0.0412\sqrt{DN}}}{0.05\times 0.174\times 0.37}} \Rightarrow 312.7 MPa$$

μ_Q = μ_{Sy}−μ_σ = 350-312.7=37.3MPa and $\sigma_Q = \sqrt{\sigma_s^2+\sigma_\sigma^2}$ =44.03MPa. The value of 'Z' is estimated by equation (8)

$$Z = \frac{Q-\mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} = -\frac{37.3}{44.03} = -0.86$$

The reliability value estimated by using the reliability table (Budynas and Nisbett, 2014) is 0.806 (=1-0.194)for the value of Z = -0.86.

From the above discussion, the gear will be functioning safe as the FOS value 1.03>1 but based on the probabilistic method the reliability is 0.806, which is low. This shows the

Importance of the considering the statistical variation in the gear design.

To improve the reliability of the gear design, it is necessary to study the sensitivity of the each variable and reduce the variation in the variable. The sensitivity of variable T, N, D and F is calculated by substituting in equation (20):

$$\sigma_{ci} = \sqrt{\left(\frac{\partial \sigma_c}{\partial x}\right)^2 \partial x_i^2}$$

$$\sigma_{cT} = \sqrt{\left(\frac{\partial \sigma_c}{\partial T}\right)^2 \partial T^2} = \sqrt{\left(1.38\times10^5\times300\right)^2} = 4.14\times10^8$$

$$\sigma_{cN} = \sqrt{\left(\frac{\partial \sigma_c}{\partial N}\right)^2 \partial N^2} = \sqrt{\left(4.65\times10^3\times100\right)^2} = 4.65\times10^5$$

$$\sigma_{cD} = \sqrt{\left(\frac{\partial \sigma_c}{\partial D}\right)^2 \partial D^2} = \sqrt{\left(1.38\times10^9\times0.5\times10^{-3}\right)^2} = 0.65\times10^6$$

$$\sigma_{cF} = \sqrt{\left(\frac{\partial \sigma_c}{\partial F}\right)^2 \partial F^2} = \sqrt{\left(3.127\times10^9\times2\times10^{-3}\right)^2} = 6.25\times10^6$$

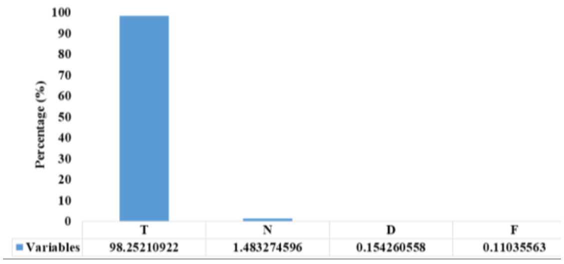


Fig.6 Sensitivity of variables

The obtained sensitivity is shown in figure 6. From this figure it can be concluded that the major variation is due to torque. To improve the reliability the standard deviation of the torque value is reduced to 100Nm and the estimated value of σ_Q is 22.55. The estimated value of Z is 1.65 and from the reliability value estimated to be 95.05. The reliability can be increased by reducing the variability in the in the dimensions of diameter and face width.

Conclusion

AGMA (American Gear Manufacturing Association) approach to design gears requires a number of subjective design correction factors. In this paper, an approach accounting mean and standard deviation of each design variable/parameter has been suggested. The proposed probabilistic approach reduces the subjectivity by eliminating correction design factors and provides more realistic results. A case study to illustrate the procedure to using AGMA and the proposed statistical approach has been considered. In the case study it was observed that the as per AGMA approach the gear was well designed but the based on the probabilistic method, due to high variability in variables the gears had low reliability. As the deviations in the torque value was reduced, the reliability of the gear has increased but the AGMA value remained same. It can be said that the proposed approach is a robust approach and shall be used to design various machine elements.

References

Hirani, H., Athre, K., Biswas, S. (1998), Rapid and Globally Convergent Method for Dynamically Loaded Journal Bearing Design, *Proc. IMechE (UK), Journal of Engineering Tribology*, vol. 212, pp. 207-214.

Hirani, H., Athre, K., Biswas, S. (1999), Dynamic Analysis of Engine Bearings, *International Journal of Rotating Machinery*, vol. 5, no.4, pp. 283-293.

Hirani, H., Athre, K., Biswas, S. (2000), A Hybrid Solution Scheme for Performance Evaluation of Crankshaft Bearings, *Trans. ASME, Journal of Tribology*, vol. 122, no. 4, pp. 733-740

Hirani, H., Athre, K., Biswas, S. (2001), A Simplified Mass Conserving Algorithm for Journal Bearing under Dynamic Loads, *International Journal of Rotating Machinery*, vol. 1, pp. 41-51.

Hirani, H., Athre, K., Biswas, S. (2001), Lubricant Shear Thinning Analysis of Engine Journal Bearings, *STLE, Journal of Tribology Transaction*, vol 44, no. 1, pp 125-131.

Muzakkir, S.M., Hirani, H., Thakre, G.D., Tyagi, M.R. (2011), Tribological Failure Analysis of Journal Bearings used in Sugar Mill, *Engineering Failure Analysis*, vol. 18, no. 8, pp. 2093-2103.

Hirani, H., Athre, K., Biswas, S.(2002), Comprehensive Design Methodology for Engine Journal Bearing, *IMechE (UK), Part J, Journal of Engineering Tribology*, vol 214, pp. 401-412.

Hirani, H.(2004), Multi-objective Optimization of a journal bearing using the Pareto optimal concept, *Proc. Institute Mech. Engineers., Part J, Journal of Engineering Tribology*, vol. 218, no. 4, pp. 323-336.

Hirani, H.(2005), Multiobjective optimization of journal bearing using mass conserving and genetic algorithms*, Proc. Institute Mech. Engineers., Part J, Journal of Engineering Tribology*, vol. 219, no. 3, pp. 235-248.

Hirani, H.,& Suh, N. P. (2005), Journal Bearing Design using Multi-objective Genetic Algorithm and Axiomatic Design Approaches, *Tribology International*, vol. 38, no. 5, pp. 481-491.

Hirani, H.,& Verma, M. (2009), Tribological study of elastomeric bearings of marine shaft system, *Tribology International*, vol. 42, no. 2, pp. 378-390.

Muzakkir, S.M., Hirani, H., Thakre, G.D., (2013), Lubricant for Heavily-Loaded Slow Speed Journal Bearing, Tribology Transactions, vol.56, no. 6, pp. 1060-1068.

Muzakkir, S.M., Lijesh, K. P., Hirani, H., Thakre, G.D. (2015) Effect of Cylindricity on the Tribological Performance of Heavily-Loaded Slow Speed Journal Bearing, Proc. Institute Mech. Engineers., *Part J, Journal of Engineering Tribology*, 2015, vol 229, no.2, pp.178-195.

Muzakkir, S.M. & Hirani, H. (2015), A Magnetorheological Fluid Based Design Of Variable Valve Timing System For Internal Combustion Engine Using Axiomatic Design, International Journal of Current Engineering Research, Vol.5, No.2 (April 2015), pp 603-612.

Rao, T. V. V. L. N., Hirani, H., Athre, K., Biswas, S.(2000), An Analytical Approach to Evaluate Dynamic Coefficients and Non-linear Transient Analysis of a Hydrodynamic Journal Bearing, *STLE Tribology Transactions*, vol. 23, no.1, pp. 109-115.

Muzakkir, S. M., K. P. Lijesh, and Harish Hirani. (2015).Effect of Base Oil on the Anti-Wear Performance of Multi-Walled Carbon Nano-tubes (MWCNT), *International Journal of Current Engineering and Technology*, vol.5, no.2, pp.681-684.

R. K. Burla, P. Seshu, H. Hirani, P.R. Sajanpawar, and H.S. Suresh ,(2004), Three Dimensional Finite Element Analysis of Crankshaft Torsional Vibrations using Parametric Modeling Techniques, *SAE transactions*, SAE 2003-01-2711.

Lijesh, K. P., Hirani, H.,(2014), Stiffness and Damping Coefficients for Rubber mounted Hybrid Bearing, *Lubrication Science*, vol. 26, no.5, pp. 301-314.

Lijesh, K. P., Hirani, H.,(2015), Development of Analytical Equations for Design and Optimization of Axially Polarized Radial Passive Magnetic Bearing, *ASME, Journal of Tribology*, vol. 137, no. 1, (9 pages).

Lijesh, K. P., Hirani, H.,(2015), Optimization of Eight Pole Radial Active Magnetic Bearing, *ASME, Journal of Tribology*, vol.137, no.2.

Lijesh, K. P.& Hirani, H. (2015), Design and Development of Halbach Electromagnet for Active Magnetic Bearing, *Progress In Electromagnetics Research C*, vol. 56, 173–181.

Desai C., Hirani H. and Chawla A., (2014), Life Estimation of Hip Joint Prosthesis, Journal of the Institution of Engineers (India): Series C, 1-7. DOI: 10.1007/s40032-014-0159-4

Lijesh K.P. and Harish Hirani, (2015), Probabilistic Design of Spur Gear, *IJCET,* Vol.5, No.2, pp. 1125-1147, E-ISSN 2277 – 4106, P-ISSN 2347 – 5161

Muzakkir SM and Hirani H, (2015), Innovative-Tribo-Design of Variable Valve Timing Mechanism: Need and Concept, *IJCET,* Vol.5, No.2, pp. 1115-1125, E-ISSN 2277 – 4106, P-ISSN 2347 – 5161.

Muzakkir, S.M., Lijesh, K. P.& Hirani, H. (2014), Tribological Failure Analysis of a Heavily-Loaded Slow Speed Hybrid Journal Bearing, *Engineering Failure Analysis*, vol. 40, pp.97–113.

Samanta, P., Hirani, H., (2007), A Simplified Optimization Approach for Permanent Magnetic Journal Bearing, *Indian Journal of Tribology*.

Lijesh K.P. and Harish Hirani, (2015), Magnetic Bearing Using RMD Configuration, *ASME, Journal of Tribology,* Accepted.

Vikas M., and Hirani H., (2006), Switching Amplifier for Frictionless Bearing, International Conference on Industrial Tribology.

Samanta, Pranab, and Harish Hirani. (2008), Magnetic bearing configurations: Theoretical and experimental studies. Magnetics, IEEE Transactions on vol. 44, no. 2, pp. 292-300.

Shankar, S., Sandeep, Hirani, H. (2006), Active Magnetic Bearing, *Indian Journal of Tribology*, pp 15-25.

Hirani, H., Samanta, P. (2007), Hybrid (Hydrodynamic + Permanent Magnetic) Journal Bearings, Proc. Institute Mech. Engineers., *Part J, Journal of Engineering Tribology*, vol. 221, no. J8, pp. 881-891.

Lijesh, K. P., and Hirani, H. (2014). Design of eight pole radial active magnetic bearing using monotonicity. In Industrial and Information Systems (ICIIS), 2014 9th International Conference on IEEE, pp. 1-6.

Lijesh K.P., Harish Hirani, Samanta P., (2015), Theoretical and Experimental Study for Hybrid Journal Bearing, International Journal of Scientific & Engineering Research, vol 6, No. 2, pp. 133-139

H. Hirani and S.S. Goilkar, (2011),Rotordynamic Analysis of Carbon Graphite Seals of a Steam Rotary Joint, *Book on IUTAM Symposium on Emerging Trends in Rotor Dynamics*, 253-262, Springer Netherlands.

S S Goilkar and H Hirani, (2010),Parametric Study on Balance Ratio of Mechanical Face Seal in Steam Environment, *Tribology International*, vol. 43, no. 5-6, pp. 1180-1185.

Hirani H and Goilkar S S, (2009),Formation of Transfer Layer and its Effect on Friction and Wear of Carbon-Graphite Face Seal under Dry, Water and Steam Environments, *Wear*, vol. 226, no. 11-12, pp.1141-1154.

Goilkar S S, and Hirani H., (2009) ,Design and development of test setup for online wear monitoring of mechanical face seals using torque sensor, *Tribology Transactions*, vol. 52, no.1, pp. 47-58.

S S Goilkar and H Hirani, (2009), Tribological Characterization of Carbon Graphite Secondary Seal *Indian Journal of Tribology*, vol. 4, no. 2, pp. 1-6.

C Sarkar and H Hirani, Development of magnetorheological brake with slotted disc, *Proc. IMechE, Part D: Journal of Automobile Engineering*, DOI: 10.1177/0954407015574204.

C Sarkar and H Hirani, (2013),Theoretical and experimental studies on a magnetorheological brake operating under compression plus shear mode, *Smart Materials and Structures*, vol.22. no.11, art. no. 115032.

C Sarkar and H Hirani, (2013), Synthesis and Characterization of Antifriction Magnetorheological Fluids for Brake, *Defense Science Journal*, vol. 63, no.4, pp.408-412.

C Sarkar and H Hirani, (2013), Design of a Squeeze Film Magnetorheological Brake Considering Compression Enhanced Shear Yield Stress of Magnetorheological Fluid, *Journal of Physics: Conference Series*, vol.412, no.1, 012045.

V K Sukhwani and H Hirani, (2008),A Comparative Study of Magnetorheological-Fluid-Brake and Magnetorheological-Grease-Brake, *Tribology Online*, vol 3, no.1, pp. 31-35.

V K Sukhwani, and H Hirani, (2008),Design, Development and Performance Evaluation of High Speed MR Brake, *Proc. Institute Mech. Engineers.,Part L, Journal of Materials: Design and Applications*, vol. 222, no.1, pp. 73-82.

H Hirani, and C S Manjunatha, (2007),Performance Evaluation of Magnetorheological Fluid Variable Valve, *Proc. of the Institution of Mechanical Engineers, Part D, Journal of Automobile Engineering*, vol. 221, no.1, pp. 83-93.

H. Shah and H Hirani, (2014),Online Condition Monitoring of Spur Gear, *International Journal of Condition Monitoring*, vol 4, no. 1, pp.15-22.

H Hirani, (2009), Online Wear Monitoring of Spur Gear *Indian Journal of Tribology*, vol 4, no.2, pp.38-43.

Al-Tubi, I., Long, H., Tavner, P., Shaw, B., & Zhang, J. (2015). Probabilistic analysis of gear flank micro-pitting risk in wind turbine gearbox using supervisory control and data acquisition data. IET Renewable Power Generation. pp. 1-5

Gautham, B. P., Gupta, P., Kulkarni, N. H., Panchal, J. H., Allen, J. K., & Mistree, F. (2013). Robust Design of Gears with Material and Load Uncertainties. In ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. V03BT03A046-V03BT03A04.

R. G. Budynas and J.K. Nisbett, (2014), Mechanical Engineering Design, Indian dition,The McGraw-Hill Companies, Inc., New York

K.Gopinath and M.M.Mayuram, NPTEL, Lecture 8, Spur Gears, http://nptel.ac.in/courses/IIT-MADRAS/Machine_Design_II/pdf/2_8.pdf.

R. L. Norton, (2001), Machine Design, An integrated approach Second edition, Addison Wesley Longman, Singapore.

SK Talluri, H Hirani, (2003), Parameter Optimization Of Journal Bearing Using Genetic Algorithm, Indian Journal of Tribology 2 (1-2), pp. 7-21.