

Research Article

# Design of Spur Gear Considering Contact Stress using Probabilistic Approach

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Accepted 15 April 2015, Available online 22 April 2015, Vol.5, No.2 (April 2015)

## Abstract

Gears are known for effective transferring of power from one shaft to another by maintaining a definite velocity ratio. Classical method AGMA (American Gear Manufacturing Association) standards are followed to design gears to ascertain the constant transmission of torque and angular speeds of shafts. In reality torque, rotational speed, gear material properties, shaft-alignment and gear geometry are statistical in nature. To account such variability in operating and geometric conditions, the AGMA standards recommend usage of a large number of empirical design factors. Due to subjective nature of those factors, often wrong input design factors lead to over-design or under-design of the gear. To decide the appropriate values of correction factors, it is advisable to account statistical variations in operating and geometric conditions in the form of their mean and standard deviation values. In the present work, a methodology of probabilistic design for durability of spur gear has been presented accounting pitting failure. A case study has been considered to compare the procedure suggested by AGMA standard and the proposed methodology.

**Keywords:** Reliability, gear, statistical variation, AGMA.

## 1. Introduction

There is a need of mechanical contact between operating gears to transmit the required torque or to operate the shafts at desired speeds. For quiet and vibration less operation of such mechanical contacts, the velocities of the pitch circles of the two gears must be same at all times. An appropriate design methodology is essential to design such gears operating under variable load, speed, and geometric conditions.

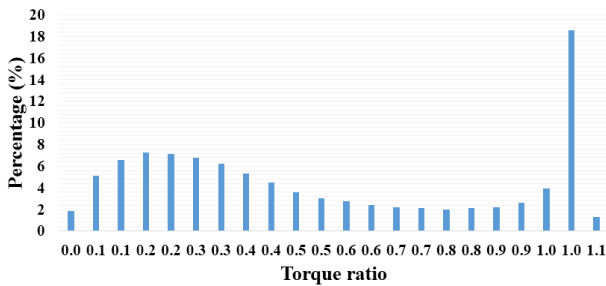
Gears are designed to resist bending failure of the teeth and pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength. It was observed by Gautham (2013) that contact factor of safety is more critical than the bending factor of safety. The contact stresses are generally higher. The gears eventually fail to function due to excessive contact stresses and therefore durability of gears is related to contact stresses. Hence in the present paper the surface pitting of gears has been dealt. The pitting is surface fatigue phenomenon caused by stresses exceeding the endurance strength (corrected endurance limit) of the gear material.

The pitting failure is related to the contact stress. Various formulae for the pitting stress are provided in standards of AGMA (American Gear Manufacturing Association). The AGMA based contact stress equation incorporates a number of correction factors such as overall load factor, dynamic factor, load distribution factor, surface condition factor etc. The values/expression of these factors are based on large numbers of tests and many years of experience and do not consider the actual uncertainties in the input data, make these factors less robust. Often the values of design factors, decided by designers, are very subjective.

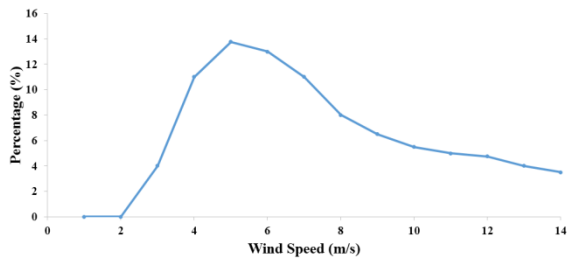
Even though AGMA has provided design guidelines, proper gear design is a challenging task and it becomes more challenging in case of fluctuating operating conditions, variable material properties and statistical geometric dimensions. Variation in operating conditions in one of typical gearbox used in wind-power-generator is presented in figure 1. There is a need of statistical design approach to deal with such variations in operating conditions. It is interesting to note that statistical variation in operating conditions is very common in most of mechanical components such as bearings [Hirani, 2009, Hirani *et al*, 2000, Hirani *et al*, 1999, Hirani *et al*, 1998, Muzakkir *et al*, 2011, Hirani, 2005, Hirani *et al*, 2001, Muzakkir *et al*, 2013, Hirani, 2004, Muzakkir *et al*, 2015, Hirani, Verma, 2009, Hirani, Suh, 2005, Hirani *et al*, 2001, Rao *et al*, 2000, Hirani *et*

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al, 2000, Hirani et al, 2002, Burla et al, 2004, Hirani, Samanta, 2007, Lijesh, Hirani, 2015, Lijesh, Hirani, 2014, Shankar et al, 2006, Lijesh, Hirani, 2015, Muzakkir et al, 2014, Lijesh, Hirani, 2015], seals [Hirani and Goilkar, 2011, Goilkar and Hirani, 2010, Hirani and Goilkar, 2009, Goilkar and Hirani, 2009, Goilkar and Hirani, 2009], brakes [Sarkar and Hirani, 2015, Sarkar and Hirani, 2015, Sarkar and Hirani, 2013, Sarkar and Hirani, 2013, Sukhwani et al, 2009, Sukhwan and Hirani, 2008, Sukhwani et al, 2008, Sukhwani et al, 2008, Hirani and Manjunath, 2007, Sukhwani et al, 2007], gears [Shah and Hirani, 2014, Hirani, 2009]. The proposed probabilistic design approach can be applied to any of aforementioned machine component.



(a) Torque ratio variation



(b) Wind Speed variation

**Fig. 1** Torque and wind speed variation of wind turbine (Issa et al 2014)

There is need to develop a design methodology to incorporate the statistical variation in load, speed, material properties, and gear dimensions. Implementation of such methodology will eliminate the need of subjective-correction factors advised by AGMA. In the present work, a methodology based on mean and standard deviation values of design factors has been presented. To exemplify the proposed design procedure, a case-study considering is presented. Each step has been described. The obtained results are matched with results obtained from AGMA procedure.

**2. Gear Design**

The designer involved with gears is expected to follow the pertinent standards of the AGMA (classical) approach using involute gearing profile. AGMA standard 2001-B88 assumes following related to mesh geometry (Norton, 2001):

- i. Contact ratio is between 1 and 2.

- ii. There is no interference between the tips and root fillets of mating teeth.
- iii. No undercutting.
- iv. No tooth is pointed
- v. There is nonzero backlash.
- vi. The root fillets are standard, assumed smooth and produced by a generating process.
- vii. The friction forces are neglected.

In the present work, a summary of AGMA approach has been presented. To design gears, the proposed probabilistic approach has been detailed. A case study to compare the AGMA and the proposed design approaches presented.

(i) Classical approach (Budynas and Nisbett, 2014)

The contact stress given by AGMA approach is given by equation (1).

$$\sigma_c = C_p \sqrt{\frac{W_t K_v K_o K_s K_H C_f}{F d I}} \tag{1}$$

Here;  $W_t$  is tangential load (N),  $F$  is face-width,  $m$  is gear-module and  $K_v$  is dynamic factor.  $K_v$  is represented as:

$$K_v = \frac{3.05 + V}{3.05} \quad \text{(Cast iron, cast profile)} \tag{2(a)}$$

$$K_v = \frac{6.1 + V}{6.1} \quad \text{(Cut or milled profile)} \tag{2(b)}$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad \text{(Hopped or shaped profile)} \tag{2(c)}$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad \text{(Shaved or ground profile)} \tag{2(d)}$$

$K_o$  is the overload factor as listed in Table 1.

**Table 1** Table of overload Factor

Power Source	Uniform	Moderate shock	Heavy Shock
Uniform	1	1.25	1.75
Light Shock	1.25	1.5	2
Medium Shock	1.5	1.75	2.25

$K_s$ , size factor, as a function of gear-module, face width and geometry is given as:

$$K_s = 1.192 (F m \sqrt{Y})^{0.0535} \tag{3}$$

$K_H$ , load distribution factor, is very complicated. It accounts non-uniformity in distribution of load across the line of contact and expressed by equation (4).

$$K_H = 1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e) \tag{4}$$

Where;

$$C_{mc} = \begin{cases} 1 & \text{For uncrowned teeth} \\ 0.8 & \text{For crowned teeth} \end{cases}$$

$$C_{pf} = \begin{cases} \frac{F}{10D} - 0.025 & F < 25mm \\ \frac{F}{10D} - 0.0375 + F \cdot 4.92 \times 10^{-4} & 25mm < F < 425mm \\ \frac{F}{10D} - 0.1109 + F \cdot 8.15 \times 10^{-4} - F^2 \cdot 3.53 \times 10^{-4} & 425mm < F < 1000mm \end{cases}$$

If  $F/(10D) < 0.05$ ,  $F/(10d) = 0.05$  is used

$$C_{pm} = \begin{cases} 1 & \text{For straddle mounted pinion with } S_1 / S < 0.175 \\ 1.1 & \text{For straddle mounted pinion with } S_1 / S \geq 0.175 \end{cases}$$

S is the center distance between two bearings and  $S_1$  is the distance between the center line of the gear face and mid-point of shaft-system as shown in figure 1.

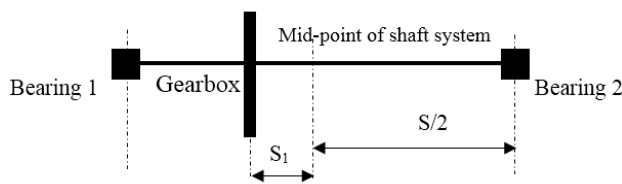


Fig.1 Definition of distance S and  $S_1$  used in evaluation

$$C_{ma} = A + BF + CF^2$$

The value of A, B, and C is given in table 2.

Table 2 Empirical Constants A, B, and C to find  $C_{ma}$

Condition	A	B	C
Open gearing	0.247	$6.57 \times 10^{-4}$	$-3.01 \times 10^{-6}$
Commercial, enclosed units	0.127	$6.22 \times 10^{-4}$	$-3.66 \times 10^{-6}$
Precision, enclosed units	0.0675	$5.03 \times 10^{-4}$	$-3.65 \times 10^{-6}$
Extra precision enclosed gear units	0.00360	$4.015 \times 10^{-4}$	$-3.24 \times 10^{-6}$

$$C_e = \begin{cases} 0.8 & \text{For gearing adjusted at assembly or compatibility is improved by lapping or both} \\ 1 & \text{For all other conditions.} \end{cases}$$

The elastic coefficient ( $C_p$ ) is given by,

$$C_p = \frac{1}{\sqrt{\pi \left[ \left( \frac{1 - \nu_p^2}{E_p} \right) + \left( \frac{1 - \nu_g^2}{E_g} \right) \right]}} \tag{5}$$

Where;  $E_p$  and  $E_g$  are the moduli of elasticity and  $\nu_p$  and  $\nu_g$  are the Poisson's ratios for pinion and gear respectively.

The surface geometry factor (I) takes into account the radii of curvature of the gear teeth and the pressure angle. As per AGMA I is given by equation (6).

$$I = \frac{\cos \phi}{\left( \frac{1}{\rho_p} \pm \frac{1}{\rho_g} \right) D} \tag{6}$$

Where;

$$\rho_p = \sqrt{\left( \frac{D}{2} + (1 + x_p)m \right)^2 - \left( \frac{D}{2} \cos \phi \right)^2} - m \pi \cos \phi \tag{7(a)}$$

$$\rho_g = C \sin \phi \mp \rho_p \tag{7(b)}$$

Where D is the pitch diameter of the pinion,  $\phi$  is the pressure angle, and C is the center distance between pinion and gear. The negative sign in equation 7(b) is for the external gear-set, while positive sign is for the internal gear-set.  $C_f$  is the surface condition factor.

To avoid pitting failure, equation (8) must be satisfied

$$\sigma_c \leq \frac{S_c Z_N Z_W}{K_T K_R S_{Fc}} \tag{8}$$

Where;  $S_c$  is the allowable contact stress based on hardness value,  $Z_N$  is the stress cycle factor,  $Z_W$  are the hardness ratio factor for pitting resistance,  $K_T$  is the temperature factor,  $K_R$  is the reliability factor and  $S_{Fc}$  is the factor of safety.

The value of allowable contact stress is given by equation 9(a).

$$S_c = \begin{cases} 2.22 H_B + 200 MPa & \text{For Grade 1} \\ 2.41 H_B + 237 MPa & \text{For Grade 2} \end{cases} \tag{9(a)}$$

The stress cycle factor ( $Z_N$ ) is given by the graph provided in figures 14-15 of book of Richard, Budynas (2014). The reliability factor ( $K_R$ ) is estimated based on the required reliability (R) and the value is estimated from equation 9(b).

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 < R < 0.9999 \end{cases} \tag{9(b)}$$

For oil or gear blank temperature up to 120°C,  $K_T$  is considered to be unity. For higher temperature the factor should be greater than unity.  $Z_W$  is calculated by equation 9(c) or 9(d). For through-hardened pinions running against through hardened gears.

$$Z_w = 1 + A(m_G - 1) \tag{9(c)}$$

Where  $m_G$  is the gear ratio, A is the calculated for following condition.

$$\begin{cases} \text{if } \frac{HB_p}{HB_g} < 1.2 \text{ then } A = 0 \\ \text{if } 1.2 < \frac{HB_p}{HB_g} < 1.7 \text{ then } A = 0.00898 \frac{HB_p}{HB_g} - 0.00829 \\ \text{if } \frac{HB_p}{HB_g} > 1.7 \text{ then } A = 0.00698 \end{cases}$$

For surface-hardened pinions (>48HRC) run against hardened gears,  $Z_w$  is found from:

$$Z_w = 1 + B(450 - H_g) \quad 9(d)$$

$$B = 0.00075e^{-0.052R_q}$$

Where,  $R_q$  is the rms surface roughness of the pinion teeth in  $\mu\text{m}$ .

### (ii) Proposed Probabilistic Design Approach

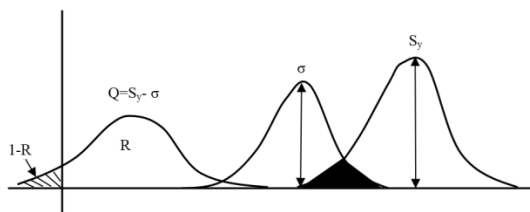
To overcome the problems related to classical approach, probabilistic approach incorporating the statistical variation in various design factors has been used. In this approach, it is assumed that the standard deviation of each variable follows the Gaussian distribution, such as:

$$f(Q) = \frac{1}{\hat{\sigma}_Q \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{Q - \mu_Q}{\hat{\sigma}_Q} \right)^2 \right] \quad (10)$$

Here  $\mu_Q$  is mean and  $\hat{\sigma}_Q$  is standard deviation of the variable concerned. To normalize the Eq. (10), concept of 'normal variable' (Equation 11), having a mean of zero and a standard deviation of unity, can be used.

$$Z = \frac{Q - \mu_Q}{\hat{\sigma}_Q} \quad (11)$$

The probability distribution curves of stress ( $\sigma$ ) and strength ( $S_y$ ) have been shown in figure 4. The mechanical component will not fail if the strength is greater than the stress. In other words, the margin of safety for any value of stress  $\sigma$  and strength  $S_y$  is defined as  $Q = S_y - \sigma$ . Any chance of  $Q < 0$ , results in the probability of component failure. The reliability (R) that a part will perform without failure is the area of the margin of safety distribution for  $Q > 0$ . The interference is the area  $(1-R)$  where part fails.



**Fig. 4** Stress and strength distributions

To find the chance that  $Q > 0$ , the value of 'Z' variable (equation 11) is obtained for  $Q=0$ .

$$Z = \frac{Q - \mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} \quad (12)$$

Where  $\mu_Q = \mu_S - \mu_\sigma$  and  $\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\sigma^2}$ .

From the value of Z, the reliability of the component is estimated using the table given in A-10 of book authored by Budynas, Nisbett (2014).

In the proposed approach, the mean and standard deviation of the induced stress is calculated. To find the mean value of stress, mean values of all design variables is considered. To account standard deviation of stress, stress equation is differentiated w.r.t. each individual independent variable. If the design variable is dependent then it is substituted as function of independent variables. To understand the procedure, consider the equation (1).

$$\sigma_c = C_p \sqrt{\frac{W_t K_v K_o K_s K_H C_f}{F d I}} \quad (1)$$

Here  $K_v$  is function of velocity, which is function of pitch diameter and rotational speed, therefore suitable expression of  $K_v$  from Eq. (2) shall be substituted in Eq. (1). Similarly  $I$  is a function of pressure angle  $\phi$ , as expressed in Eq. (6). Interesting the running pressure angle is function of operating distance between centers of gears. During assembly there is a possibility of variation in center distance, due to which nominal pitch circle diameter and pressure angle ( $\phi$ ) vary. As base pitch remains constant, pressure angle can be expressed as:

$$\phi = \cos^{-1} \left( \frac{r_{p \text{ nom}} \{ \cos \phi_{\text{nom}} \}}{r_p} \right) \quad 13(a)$$

'nom' is the suffix for 'nominal'. Pitch circle radius ( $r_p$ ) can be replaced in terms of pitch circle diameter (D), such as:

$$\phi = \cos^{-1} \left( \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{D} \right) \quad 13(b)$$

$D_{\text{nom}}$  is given as module \* number\_of\_teeth

$$\phi = \cos^{-1} \left( \frac{m Z_p \cos \phi_{\text{nom}}}{D} \right) \quad 13(c)$$

This expression of pressure angle can be substituted in expression of 'T' as module (m), number of teeth ( $Z_p$ ) and nominal pressure angle ( $\phi_{\text{nom}}$ ) remain constant. Once these parameters are selected, their values remain constant.

Similarly, substituting values in Eq. (7)

$$\rho_p = \sqrt{\left( \frac{D}{2} + m \right)^2 - \left( \frac{D}{2} \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{D} \right)^2} - m \pi \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{D}$$

$$\rho_p = \sqrt{\left( \frac{D}{2} + m \right)^2 - \left( \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{2} \right)^2} - m \pi \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{D} \quad 13(d)$$

Centre distance can be represented in diameter of pinion (D) and gear ratio ( $m_g$ ) as given in equation 7(d)

$$\rho_g = C \sin \phi - \rho_p$$

$$\rho_g = \frac{D(1 + m_g)}{2} \sin \left( \cos^{-1} \left( \frac{D_{\text{nom}} \cos \phi_{\text{nom}}}{D} \right) \right) -$$

$$\sqrt{\left( \frac{D}{2} + m \right)^2 - \left( \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{2} \right)^2} + m \pi \frac{D_{\text{nom}} \{ \cos \phi_{\text{nom}} \}}{D} \quad 13(e)$$

Substituting equations 13(d) and 13(e) in equation (6) for external gear we get;

$$I = \frac{\frac{D_{nom}\{\cos\phi_{nom}\}}{D}}{\left(\frac{\rho_g + \rho_p}{\rho_p\rho_g}\right)D} \Rightarrow \frac{D_{nom}\{\cos\phi_{nom}\}}{\left(\frac{D^3(1+m_g)D_{nom}\{\cos\phi_{nom}\}}{\rho_p\rho_g}\right)} \Rightarrow \frac{\rho_p\rho_g}{D^2(1+m_g)}$$

$$I = \frac{\left[ \frac{D(1+m_g)}{2} \sin\left(\cos^{-1}\left(\frac{mZ_p \cos\phi_{nom}}{D}\right)\right) \times \left[ \sqrt{\left(\frac{D}{2} + m\right)^2 - \left(\frac{D_{nom}\{\cos\phi_{nom}\}}{2}\right)^2} - m\pi \frac{D_{nom}\{\cos\phi_{nom}\}}{D} \right] \right]}{D^2(1+m_g)} \quad 13(f)$$

The stress equation given in equation (1) is can be rewritten as a function of  $K_v, W_t, F, K_o, K_s, K_H, C_f, D, I$  and  $C_p$  as given in equation (14).

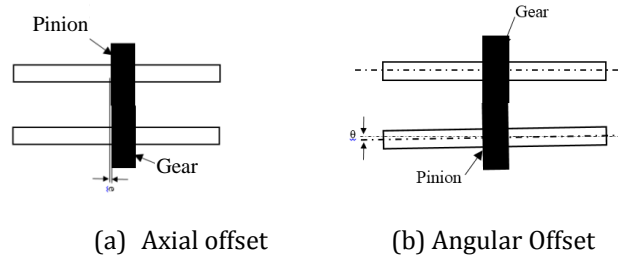
$$\sigma_c = f(K_v, W_t, F, K_o, K_s, K_H, C_f, D, I, C_p) \quad (14)$$

The justification of considering the variables as dependent and independent is given below:

- (i)The variable load ( $W_t$ ) is dependent on the applied torque (T) and pitch diameter (D) ( $W_t=2T/D$ ). Therefore  $f(W_t)$  is replaced by  $f(T,D)$ .
- (ii) $K_v$  is function pitch line velocity (V) (given equation (2)) and V is function of angular speed (N) and pitch diameter (D) ( $V=\pi DN/60$ ).
- (iii)In the present approach variable in load is accounted by considering variations in torque and speed; therefore there is no need to consider the over load factor ( $K_o$ ).
- (iv)The load distribution factor ( $K_H$ ) is the due to the improper engagement of gear and pinion as shown in figure 5. Due to the assembly error in engaging the gear and pinion, as well in the mounting of bearings, the faces of gear and pinion shift and as a result effective face width in contact decreases.

It is interesting to note that gear effective width depends on the alignment and rigidity of the support bearings. Figure 5(a) depicts an axial offset between the gears, which results in reduction in contacting face width. Similarly an angular offset is shown in figure 5(b). These offsets arise due to the clearance in the bearings and coupling (i.e. Jaw coupling) between driving and driven shafts. Therefore to incorporate the design factor for load distribution, the full assembly has to be considered. This problem can be overwhelmed by including the statistical variation considering the full system. However to get the accurate results, intensive study has to be performed which is out of scope of the present work. In statistical approach, this can be accounted by consider the minimum value of face width. To

understand this let us consider mean value of width as  $\mu_b$  and standard deviation as  $\sigma_b$ . In statistical approach, the minimum value of face width will be  $\mu_b-3\sigma_b$ . By considering this, there is no need to account  $K_H$  and standard deviation in face width. From equation (4)  $K_H=f(F,D)$ .



**Fig. 5** axial and angular offset during gear and pinion engagement

- (v)In statistical approach variation in material properties is accounted. In addition, the minimum possible value of face width is considered. Therefore, there is no need to account size factor  $K_s$ .
- (vi)There is very rare chance of designing thin rim of gear. Generally  $t_r$  is far greater than the tooth thickness ( $m_B>1.2$ ). The basic reason for stating  $m_B>1.2$  is that the diameter of shaft is generally much smaller than the pitch diameter of the gear. Therefore, there is no need to account  $K_B$ .
- (vii)In equation 7(e), the value of  $m_g, Z_p$  and  $P_c$  are constant for any type of gear system, hence 'I' is the function of diameter (D). As the Young's modulus and poisson's ratio will be remaining constant the value of  $C_p$  shall be considered as constant.

From the above discussion, equation (14) is reduced to equation (15).

$$\sigma_c = f(T, F, D, N) \quad (15)$$

$$\sigma_c = \sqrt{\frac{1}{\pi \left[ \left( \frac{1-\nu_p^2}{E_p} \right) + \left( \frac{1-\nu_g^2}{E_g} \right) \right]}} \sqrt{\frac{Tf(D,N)f(F,D)}{FD^2f(D)}} \quad (16)$$

The standard deviation of equation (16) is estimated by equation (17)

$$\sigma_c = \sqrt{\sum_{i=1}^n \left( \frac{\partial \sigma_c}{\partial x_i} \right)^2 \partial x_i^2} \quad (17)$$

$$\sigma_c = \sqrt{\left( \frac{\partial \sigma_c}{\partial D} \right)^2 \sigma_D^2 + \left( \frac{\partial \sigma_c}{\partial T} \right)^2 \sigma_T^2 + \left( \frac{\partial \sigma_c}{\partial F} \right)^2 \sigma_F^2 + \left( \frac{\partial \sigma_c}{\partial N} \right)^2 \sigma_N^2} \quad (18)$$

The mean value ( $\mu_\sigma$ ) of stress is obtained by substituting the mean value of each variable given in equation (1).

To examine the variation of solutions attained using with and without statistical variation of AGMA equation (1), a case study is considered and comparison of results is presented.

**Case Study:** Determine durability of a high quality (shaved and grounded) pinion of spur gear system to transmit a torque of 1130Nm @ 4500 rpm with speed reduction of 3.5. Assume the pressure angle = 20°, number of teeth ( $Z_p$ )=29 module(m)=6mm. The corrected endurance strength of gear material varies in the range of 300 MPa to 400 MPa. Standard deviations in torque is  $\sigma_T = 300$  Nm, in speed 100 rpm and in pitch diameter 0.5mm. Due to crowning and manufacturing errors, the face width varies from 50mm to 52mm. Due to possibilities of angular and parallel offset, effective face width may varies in the range of 40mm to 52mm. Considered the gear is made of steel material (210GPa).

**Solution:** To determine the durability of spur gear, one of the two approaches (i) AGMA classical approach and (ii) Proposed probabilistic approach can be used. In the present case, both the approaches have been used and comparison among the obtained results has been presented.

### (i) Classical Approach

The values of correction factors:

$$K_v = \sqrt{1 + 0.0412\sqrt{DN}}$$

$$K_s = 1.192(F_{min}m\sqrt{Y})^{0.0535}$$

$$\Rightarrow 1.192(0.05 \times 0.006 \times \sqrt{0.357})^{0.0535} \Rightarrow 0.7513 < 1 \Rightarrow K_s = 1$$

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \Rightarrow 1.135$$

The stress calculated using the AGMA equation given in equation (1) is estimated as follows:

$$\sigma_c = C_p \sqrt{W_t K_v K_o K_s K_H \frac{C_f}{FDI}}$$

$$\sigma_c = 1.91 \times 10^5 \sqrt{\frac{1130}{0.174} \frac{\sqrt{1 + 0.0412\sqrt{DN}} \times 1.25 \times 1 \times 1.13 \times 1}{0.051 \times 0.174 \times 0.1201}}$$

$$\Rightarrow \sigma_c = 678 \text{ MPa}$$

$$FOS = \frac{(800 + 600)}{2\sigma_Q} = 1.03$$

FOS of safety is 1.03.

### (ii) Probabilistic Approach

As per provided information, the dynamic factor for shaved and grounded pinion is given as:

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad (19)$$

The pitch line velocity (V) is a function of pitch circle diameter of pinion (D) and speed (N). On incorporating these values, equation (16) is rewritten as:

$$K_v = \sqrt{\frac{5.56 + \sqrt{\frac{\pi DN}{60}}}{5.56}} \Rightarrow \sqrt{1 + 0.0412\sqrt{DN}}$$

$$C_p = \frac{1}{\sqrt{\pi \left[ \left( \frac{1 - \nu_p^2}{E_p} \right) + \left( \frac{1 - \nu_g^2}{E_g} \right) \right]}} \Rightarrow 1.91 \times 10^5 Pa^{0.5}$$

$$I = \frac{\left[ (1 + m_g) D_{nom} \left( \cos \phi_{nom} \sqrt{\left( \frac{D}{2} + m \right)^2 - \left( \frac{D_{nom} \{ \cos \phi_{nom} \}}{2} \right)^2} - m \pi \frac{D_{nom} \{ \cos \phi_{nom} \}}{D} \right) \right]}{\left[ \sqrt{\left( \frac{D}{2} + m \right)^2 - \left( \frac{D_{nom} \{ \cos \phi_{nom} \}}{2} \right)^2} - m \pi \frac{D_{nom} \{ \cos \phi_{nom} \}}{D} \right]^2}$$

$$I = \frac{\left\{ 0.654 \times \left[ \sqrt{\left( \frac{D}{2} + 0.006 \right)^2 - \frac{0.0031}{D}} \right] + \left[ \sqrt{\left( \frac{D}{2} + 0.006 \right)^2 - (0.0067)^2} - \frac{0.0031}{D} \right]^2 \right\}}{4D^2} \quad (20)$$

Substituting the above estimated values in equation (21) we get:

$$\sigma_c = 1.91 \times 10^5 \sqrt{\frac{T \sqrt{1 + 0.0412\sqrt{DN}}}{FD^2 I}} \quad (21)$$

Partial differentiation equation (21) w.r.t. D, N, T and F is difficult. Therefore in the present the differentiation is performed using MATLAB software.

$$\frac{\partial \sigma_c}{\partial T} = 1.38 \times 10^5, \frac{\partial \sigma_c}{\partial N} = 4.65 \times 10^3, \frac{\partial \sigma_c}{\partial D} = -1.38 \times 10^9, \frac{\partial \sigma_c}{\partial F} = -3.127 \times 10^9$$

Given  $\sigma_T = 300$  Nm,  $\sigma_N = 100$  rpm,  $\sigma_F = 2$  mm, and  $\sigma_D = 0.5$  mm. Substituting the values obtained and given values in equation (15), the estimated value of  $\sigma_s$  is 46.97MPa. Now to calculate the value of normalized factor 'Z',  $\mu_{sy}$ ,  $\sigma_{sy}$ ,  $\mu_Q$ ,  $\sigma_Q$  is calculated as follows

$$\sigma_{sy} = (400 - 300) / 6 = 16.67 \text{ MPa},$$

$$\mu_{sy} = (400 + 300) / 2 = 350 \text{ MPa}.$$

$$\sigma_c = C_p \sqrt{\frac{W_t K_v}{FDI}} \Rightarrow 1.91 \times 10^5 \sqrt{\frac{1130}{0.174} \frac{\sqrt{1 + 0.0412\sqrt{DN}}}{0.05 \times 0.174 \times 0.37}} \Rightarrow 312.7 \text{ MPa}$$

$$\mu_Q = \mu_{sy} - \mu_\sigma = 350 - 312.7 = 37.3 \text{ MPa} \quad \text{and}$$

$\sigma_Q = \sqrt{\sigma_s^2 + \sigma_\sigma^2} = 44.03 \text{ MPa}$ . The value of 'Z' is estimated by equation (8)

$$Z = \frac{Q - \mu_Q}{\sigma_Q} = -\frac{\mu_Q}{\sigma_Q} = -\frac{37.3}{44.03} = -0.86$$

The reliability value estimated by using the reliability table (Budynas and Nisbett, 2014) is 0.806 (=1-0.194) for the value of Z = -0.86.

From the above discussion, the gear will be functioning safe as the FOS value 1.03 > 1 but based on the probabilistic method the reliability is 0.806, which is low. This shows the

Importance of the considering the statistical variation in the gear design.

To improve the reliability of the gear design, it is necessary to study the sensitivity of the each variable

and reduce the variation in the variable. The sensitivity of variable T, N, D and F is calculated by substituting in equation (20):

$$\sigma_{ci} = \sqrt{\left(\frac{\partial \sigma_c}{\partial x_i}\right)^2 \partial x_i^2}$$

$$\sigma_{cT} = \sqrt{\left(\frac{\partial \sigma_c}{\partial T}\right)^2 \partial T^2} = \sqrt{(1.38 \times 10^5 \times 300)^2} = 4.14 \times 10^8$$

$$\sigma_{cN} = \sqrt{\left(\frac{\partial \sigma_c}{\partial N}\right)^2 \partial N^2} = \sqrt{(4.65 \times 10^3 \times 100)^2} = 4.65 \times 10^5$$

$$\sigma_{cD} = \sqrt{\left(\frac{\partial \sigma_c}{\partial D}\right)^2 \partial D^2} = \sqrt{(1.38 \times 10^9 \times 0.5 \times 10^{-3})^2} = 0.65 \times 10^6$$

$$\sigma_{cF} = \sqrt{\left(\frac{\partial \sigma_c}{\partial F}\right)^2 \partial F^2} = \sqrt{(3.127 \times 10^9 \times 2 \times 10^{-3})^2} = 6.25 \times 10^6$$

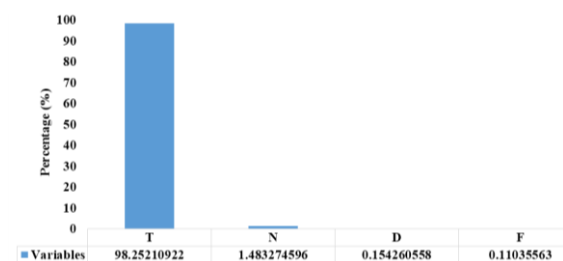


Fig.6 Sensitivity of variables

The obtained sensitivity is shown in figure 6. From this figure it can be concluded that the major variation is due to torque. To improve the reliability the standard deviation of the torque value is reduced to 100Nm and the estimated value of  $\sigma_Q$  is 22.55. The estimated value of Z is 1.65 and from the reliability value estimated to be 95.05. The reliability can be increased by reducing the variability in the in the dimensions of diameter and face width.

## Conclusion

AGMA (American Gear Manufacturing Association) approach to design gears requires a number of subjective design correction factors. In this paper, an approach accounting mean and standard deviation of each design variable/parameter has been suggested. The proposed probabilistic approach reduces the subjectivity by eliminating correction design factors and provides more realistic results. A case study to illustrate the procedure to using AGMA and the proposed statistical approach has been considered. In the case study it was observed that the as per AGMA approach the gear was well designed but the based on the probabilistic method, due to high variability in variables the gears had low reliability. As the deviations in the torque value was reduced, the reliability of the gear has increased but the AGMA value remained same. It can be said that the proposed approach is a robust approach and shall be used to design various machine elements.

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