

Research Article

Image Transformation and Compression using Fourier Transformation

S.S.Pandey[†], Manu Pratap Singh[‡] and Vikas Pandey^{*†}

[†]Department of Mathematics & Computer Science, Rani Durgawati University, Jabalpur (M. P.), India

[‡]Department of Computer Science, Institute of Engineering & Technology, Dr. B. R. Ambedkar University, Khandari, Agra (U. P.), India

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Abstract

In this paper we are discussing the Fourier transform technique for image transformation and compression. Here, we are presenting the implementation technique for image compression by using the Fourier transform method and analyzing the performance. We are also observing the results after transformation of image with quantization process. The quality of the reconstructed compressed image is also analyzed after applying the de-quantization method. The analysis of image quality is performed by using mean square of error and peak signal noise ratio. The results are showing that it is possible to achieve maximum PSNR value.

Keywords: Fast Fourier Transformation, Image compression, Image transformation, Quantization.

1. Introduction

In current era, the use of digital images has increased rapidly (A. M. Eslicioğlu & P. S. Fisher, 1995; T. Kin & X. P. Zhang 2008). The reason behind this is the storage transmission modification of these images. Image coding is a kind of method by which the data compression can achieve with the ensure quality of images and with reduce code rate. Thus by using image coding we can get the goal for saving bandwidth or space and it may also be provided for multimedia computer processing. The Fourier transform is an important image processing tool which is used to decompose in image into its sine and cosine components. The output of the transformation represents (H. B. Kekre, T. Saroode & P. Natu, 2014) the image in the Fourier or frequency domain while the input image is the spatial domain equivalent. Fourier transformation have been widely used in signal and image processing ever since the discovery of the fast Fourier transform in 1965 which made the computation of discrete Fourier transform feasible using a computer. Fast Fourier transform is on efficient implementation of the discrete Fourier transform for the discrete transformation. In the Fourier domain image each point represents a particular frequency contained. In the spatial domain image the Fourier transform is used in a wide range of application, such as image analysis image filter and image compression. The good nature of Fourier transform is that, it has a wide range of application in the image coding, image segmentation, image reconstruction and other areas.

Discrete Fourier transform with good energy concentration, due to the inconvenient operations the great amount of calculation hasn't long been widely used.

The conventional methods of image compression consider the disadvantage due to its complex algorithm and time consuming process. It has been seen and realized that with the use of fast Fourier transform (FFT) the feasibility of image compression with limited distortion in the image can investigate (K. Sahnoun & N. B. Diji, 2014; R. J. E. Merry 2005). We utilize the conjugate symmetry to reduce the data storage with reducing the time of consuming and for the different requirement of the compression ratio for better image quality. In this study we have developed a technique of image compression based on the Fast Fourier transformation for compression and decompression of image with satisfactory reconstructed image. In this paper we present the implementation of a technique for image compression by using the Fourier transform method. We apply Fourier transform method for grayscale image with different resolutions and we observe the results of transform image by quantization process. Further we apply the de-quantization method for the reconstruction of image and the quality of the reconstructed compressed image is further analyzed. The results are presented with different quantization matrix on fixed size of blocks of an image (4×4, 8×8, 16×16, 32×32). The performance of the better quality image is analyzed with maximum PSNR value and compression ratio.

*Corresponding author: Vikas Pandey

2. Fourier transform in image processing

The Fourier transform is a fundamental importance in (A. McAndrew, 2004) image processing. Fourier transform is a classical method to convert image from space domain to frequency domain and it also the foundation of image processing titled as the second language for image description. It provides another perspective for image observation and images to frequency distribution characteristics. The Fourier transform among other things provides a power for alternate to linear spatial filtering. It is more efficient to use the Fourier transform than a spatial filter. For a large filter the Fourier transform also allows us to isolate and process particular image frequencies and from low-pass and high pass with a great degree of precision. The image processing at often tends to do corresponding transformation for image by converting domain when facing to the problems that is complex and hard to deal with (J. Hu, Z. Shan, 2011).

Fourier's representation of function as a superposition of sines and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of communication signals. The Fourier transform is utilities in its ability to analyze a signal in the time domain for its frequency content. The signal can then be analyzed for its frequency content because the Fourier coefficients of the transformed function represent the contribution of each sine and cosine functions at each frequency. An inverse Fourier transform does just what you'd expect transform data from the frequency domain into the time domain.

The Fourier transform decomposes a signal into orthogonal trigonometric basis function. The Fourier transform of a continuous signal $a(x)$ can be defined as in equation (1) (A. Graphs, 1995, R. J. E. Merry, 2005; J. C. Goswami & A. K. Chan, 2011). The Fourier transformed signal $A_{ft}(y)$ gives the global frequency distribution of the time signals $a(x)$. The original signal can be reconstructed using the inverse Fourier transform as shown in equation (2).

$$A_{yx}(y) = \int_{-\infty}^{\infty} a(x)e^{-2\pi yx} dx \quad (1)$$

$$a(x) = \int_{-\infty}^{\infty} A_{yx}(y)e^{-2\pi yx} dy \quad (2)$$

Using these equations, a signal $a(x)$ can be transformed into the frequency domain and back again. The Fourier method is the most powerful technique signal analysis it transforms the signal from one domain to another domain in which many characteristics of the signal are revealed. One usually refers to this transform domain as the spectral or frequency domain, while the domain of the original signal is usually the time or spatial domain.

The Discrete Fourier Transform (DFT) is so attractive for image processing. DFT is the transform which takes the discrete signal in time domain and transforms that signal in its discrete frequency domain.

Representation this property of DFT signifies the importance of DFT in the area of spectrum analysis. There are number of extremely fast and efficient algorithms (K. Moreland & E. Angel 2003; O. Fialka & M. Cadik 2006; Y. Yoo 2001) for computing a DFT. One of the algorithms is called as fast Fourier transform or FFT. It is fast and efficient way of calculating DFT, which reduces number of arithmetical computations from $O(N)^2$ to $O(N \log_2 N)$ (W. Li, D. Zhand, & Xu. Zhuoqum 2002; Y. Yoo 2001). The key of the algorithm is data reorganization and further operations on it. The fast Fourier transform (FFT) also known's as frequency analysis or spectral involved in the implementation of many digital technique for processing signals and images (H. B. Kekre, T. Saroode & P. Natu 2014). FFT being the high speed and discrete nature equivalent of DFT is suitable for the signal's spectrum analysis in MATLAB. The FFT is a computationally efficient algorithm to compute the DFT and IDFT as shown in equation 4 and 5.

$$H(u, v) = \frac{1}{mn} \sum_{x=0}^m \sum_{y=0}^n h(x, y) e^{-j2\pi(\frac{ux}{m} + \frac{vy}{n})} \quad (4)$$

$$h(x, y) = \sum_{x=0}^m \sum_{y=0}^n H(u, v) e^{j2\pi(\frac{ux}{m} + \frac{vy}{n})} \quad (5)$$

The equations (4) and (5) are representing discrete Fourier transform (DFT) and inverse discrete Fourier transformation (IDFT) respectively. The use of an FFT vastly reduces the time needed to compute a DFT. The FFT method works recursively by dividing the original vector into two halves, computing the FFT of each half and then putting the results together. Thus the FFT is most efficient when the vector length is a power of 2 (R. C. Gonnzalez, R. E. Woods & S. L. Eddins 2009). The FFT of an image of size $m \times n$ is obtained in MATLAB by the function `fft2`. It Compute its Fourier transform And display the spectrum. This function returns a Fourier transform that is also of size $m \times n$, with the data arranged in the form, origin of the data at the top left and with four quarter periods meeting at the centre of the frequency rectangle as shown in figure 1.1 and figure 1.2



Fig.1.1 Original image

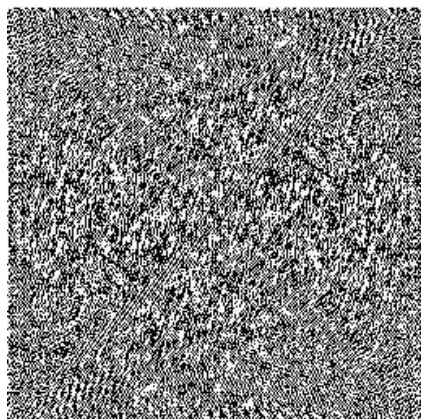


Fig.1.2 Fourier transformed image

The Fourier spectrum is obtained by using function abs i.e. $S = \text{abs}(C)$. This computes the magnitude of each element of the array. Visual analysis of the spectrum is obtained by displaying it as an image. It is an important aspect of working in the frequency domain. fft2 puts the zero frequency components at the top left corner. Another function fftshift can be used to make the origin of the transform to the centre of the frequency as shown in figure 1.3 and 1.4.



Fig.1.3 fftshift transformed image

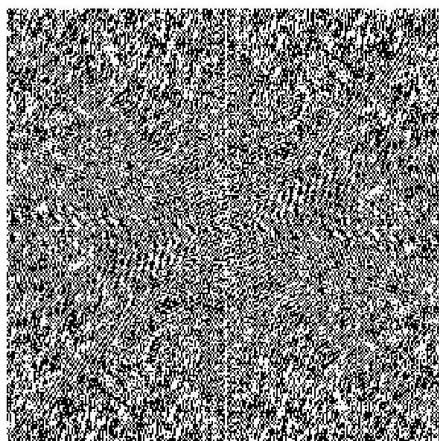


Fig.1.4 Fourier transformed apply

3. Experiment Design and Implementation Detail

In our experiment we have used the grayscale images with different size of (256×256, 512×512 & 1024×1024) resolution pixels. In this paper we have implementation and simulation design for the images transformation by applying Fourier transform and subdividing the images block size of 4×4. We increased the images blocks just like 8×8, 16×16 and the maximum blocks are used 32×32. Then after we apply the fft2 function for transforming the whole images blocks with Fourier transform and then we apply quantization method into the transformed images. Quantization is the most important step in the compression method. Its function is to map the continuous transform coefficient in limited data set.

The two dimensional Fourier transform (fft2) is computed for each block. The fft coefficients are quantized and transmitted then after the de-quantized of fft coefficients are computed. The two dimension inverse i.e. ifft2 of each block is computed and then puts the blocks back together into reconstructed images.

4. Results and Discussion

Performance of the compressed image after the transformation can be evaluated by using the following parameters:

- MSE (Mean Square Error),
- PSNR (Peak Signal Noise Ratio)
- compression ratio (CR)

These can be defined as:

$$\text{MSE} = \frac{1}{m,n} \sum \sum (X_{i,j} - Y_{i,j})^2 \quad (6)$$

Where X is original image, Y is the approximation of decompressed image and m, n are dimensions of the image.

$$\text{PSNR} = 20 * \log_{10} \left(\frac{255^2}{\text{MSE}} \right) \quad (7)$$

$$\text{CR} = \left(\frac{\text{Original image size} - \text{Reconstructed image size}}{\text{Original image size}} \right) \times 100 \quad (8)$$

The MSE and PSNR reflect the overall differences between the original image and reconstructed image.

In the proposed implementation and simulation design of the experiment, we have considered the results as shown in table I (a-d), and observed compressed image quality for gray scale images of sizes (256×256, 512×512 & 1024×1024). The images are subdivided into 4×4, 8×8, 16×16 & 32×32 blocks and transformed with fft2 . In our experiment first we chose a 256×256 resolution pixel size grayscale image.



Fig.1.5 PSNR= 43.1472db, CR= 40.55 B



Fig.1.6 PSNR= 22.4339db, CR= 71.72

Then we apply our method of block wise image compression. In which the blocks with different quantization matrices are used and then after we observed the reconstructed image as shown in figure 1.5. In this picture the quality is evaluated and it is observed that the PSNR is maximum (43.1472 db) because quantization matrix value is minimum but if we increase the quantization matrix value sequentially and applied with compression method then we can see the reconstructed image picture quality. Here the PSNR value is decreased but compression ratio are increased and with maximum CR (71.72) as shown in figure 1.6. One important point that we majored according to results of table-I, if 256×256 resolution pixel size image compress by fourier transform for image subdivided into 16×16 blocks then reconstructed image always gives best picture quality with good compression ratio.



Fig.1.8 PSNR= 29.3481db, CR= 72.37



Fig.1.7 PSNR= 36.4944db, CR=7.04

In second experiment we applied the same method and procedure for large size of image i.e. 512×512 resolution pixel image. We observed the results if compressed image quality PSNR value (43.3580 db) is maximum,when image compressed by subdivided in block 32×32. And we also seen results table-II (a to d) for observed compression ratio is maximum (83.08) when image compressed by subdivided in block 8×8.

Now applied experiment in last stage in which is applied on the largest size of image (1024×1024) for image compression. In this the image is subdivided into biggest size of block (32×32), then after we observed the reconstruct image quality in which the PSNR value is maximum (44.2071db) and compression ratio value is nearest to zero when image is compressed by subdividing the image into block size of 4×4, 16×16, 32×32 by applying minimum to maximum quantization matrix as shown in table -III (a to d). If largest image is compressed by this method then PSNR value of the reconstructed image is maximum but compression ratio value is lowest. Whereas if image is subdivided into 8×8 blocks then the PSNR value of the reconstructed image is good (33.5783 db). In this case the quantization matrix value is minimum and comparison ratio is also on the highest value i.e. 69.82. Therefore the image is blur because quantization matrix is maximum as shown in figure 1.9 and 1.10.



Fig.1.9 PSNR= 44.2071db, CR= 0.24



Fig.1.10 PSNR= 27.6016db, CR= 69.82

We can also observed one important point from the table-III i.e. if we compressed the image of large block size and we apply the proposed method then the image size does not reduce instead of this it increases. Thus, if large size of image is compressed (block 32×32) by fourier transform then it does not perform provide the good quality of the reconstructed image.

Conclusion

In this paper we have presented the method for gray scale image compression based on the Fourier transform (FFT). In this paper we have performed experiments on different size of images and then compressed the image. The transformed compressed images were reconstructed. The quality of the reconstructed images has been evaluated. Thus, we observed the results of the Fourier transform based method for image compression. The reconstructed compressed image after the transformation is obtained with good picture quality and better compression ratio. The proposed method provides better quality of the gray scale images. The simulated results on different size of images are showing superiority of Fourier transformation. It is observed that if PSNR value is less than 30 db then reconstructed compressed image is blurred and blocking effects.

The simulated results are also indicating that the FFT based image compression experiment also improved the picture quality. In this case the PSNR value becomes maximum with minimum CR and MSE values, and vice-versa. Only one disadvantage of Fourier transform is observed when the method is applied to the large size image (1024×1024) which is divided into block size of 32×32 . In this case the reconstructed compressed image takes more space with respect to the original image.

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