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Research Article

A Review on Dynamic Response Modelling of Faulted Gearbox

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Abstract

Gearbox degrades overtime due to operating conditions like load, speed, and environmental factors. The faults develop over the time. The early detection and to understand the mechanism of these are important. The current work is the consolidate the dynamic models of the gearbox to understand the behavior of the gearbox under faulted conditions.

Keywords: *Gear mesh stiffness, gear mesh damping, dynamic response*

1. Introduction

The gearbox forms an integral part of many mechanisms used and provides the size to speed and torque transmission advantage in the system. The gear elements transmit power and rotary motion by successive teeth engagement. Gear transmission has several advantages over the other transmission mechanisms in terms of overall dimensions, operational simplicity, and higher transmission efficiency. The science of gears is developing continuously with the goals to enhance the transmission life, operating efficiency, and reliability. The gearbox is used in the industrial, civilian and military application, for example in conveyor belts, wind turbines, helicopters (mining industry etc.). The gearbox operates under both constant and varying operating conditions. The gearbox consists of gears, shafts, bearings and support structure. Under the varying operating condition, the gears continue to degrade. If the gear faults are not detected at the early stage, this may be the reason for the substantial monetary and life losses [1]. In the gears the stresses are pure rolling at pitch line, above and below the pitch line rolling-sliding action takes place; the sliding is in opposite direction [2–6]. The sliding interfaces have no problem if appropriately lubricated. In insufficient lubrication surface disparities are in direct contact and differential surface hardness, rise in temperature, and adhesive bonding under high pressure contributes in breakdown of the gear surfaces[2,5]. The roots of the gear are having tension (loaded side) and compression (opposite side) at the same time. The root is the point of highest stress in tension. The bending strength of the root is the direct function of the surface hardness, surface smoothness, sharpness of radius and the fault like (crack/ pitting etc.)[2,7].

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Under these conditions the gear material is under continuous degradation and when the failure crosses the threshold called failure. The gear failures are classified based on lubrication and non-lubrication. The gear pairs work under the elastohydrodynamic lubrication and partial-elastohydrodynamic lubrication mechanism. The non-lubricated failures include the overload and bending type of failure. The lubricated failure is a set of fatigue (pitting), wear, and scuffing. The gear teeth due to deterioration generate dynamic forces which accelerate the gear tooth failure. In literature the tribological studies on the failure of gears[8–17] and bearing and synthesis of lubricant [18–69] is carried out to understand the mechanisms. Lubricant properties are determinantal for the safe working of the interfaces which are in contact. The wear is continuous process when to surfaces are in contact and threshold is depending upon the expected lifetime.[2]

Every machine element exhibits a unique vibration signature under standard conditions. The faults change the signature in the way it related to the fault. If a machine does not have any fault it generates vibrations; these vibrations relates to the periodic events of machine operation. These consists of harmonics of the rotating shaft, meshing of gear teeth, electric field etc. These harmonics are a reliable indicator of the sources and can be used as a diagnostic tool[70,71]. The vibration from different source consists of different information, to quantify the source it is necessary to understand the behaviour of the vibration output of the different sources. The vibration analysis consists of different studies such as time domain, frequency domain, and time-frequency domain [72–74].

All type of wear (pitting, mild-wear, scoring, etc.) affects the dynamics of the gearbox and is responsible for the high vibration and noise. The dynamic modelling to mimic this phenomenon of analysis of the contacting mechanism of the gear is developed, and dynamic behaviour of the system under various health condition can be simulated. The gear meshing phenomenon is highly complex and need design-based understanding of the gear system[75–77]. The different type of fault can be simulated and revelled the fault detection and diagnosis.

The present work is the consolidation of the dynamic models of the healthy and faulted gearbox.

2. Gearbox fault modelling and fault diagnosis techniques

2.1 Gearbox dynamic modelling

2.1.1 Gearbox modelling (translation and torional vibration)

All Gearbox is a complex system, and modelling of such complex system requires better understanding of the physical laws like Newtonian laws of motion (vector analysis), energy laws (Langrangian laws: scalar analysis), and laws of equilibrium so as to simulate the responses. The simulation provides the advantage of non-interference of the surrounding environment noise. Dynamic modelling summarises the stresses (contact and bending stresses), type of fault (pitting, mild-wear, scoring, and crack), transmission efficiency, loads on machine elements (like load on bearings), natural frequencies of system, response of the system, reliability, life of component, whirling of rotors[78–82]. The complexities of the gear system limit the study of simulation by imposing the restriction that all details are not modelled [83]. If it is tried to model all the details, the governing equations present more non-linearities, that are hard to solve by mathematical operations along with increase the cost of computation[84,85]. Two techniques are used to model the gear train: 1. Lumped parameter model, 2. Finite element model. In lumped parameter model the components are considered with masses concentrated at set of points. In finite element model, the masses are distributed and final response is the integration of the responses. In the initial studies, the dynamic load on gears estimated systematically during 1920s and 1930s. In 1950s, the simple spring mass system is presented to estimate the dynamic load, later the models are expended to evaluate the dynamic behaviour of gear in mesh. After 1970s the more focus area has been on estimation of gear stiffness, non-linearity of system element, damping, effects of friction in excitation, gear errors (transmission, manufacturing error), different mode of vibration, steady state and transient responses of system[86–94]. The mathematical models are grouped into following classification: (1) Simple dynamic models: estimation of the dynamic factor to estimate the stress on the tooth (2) Tooth compliance model: these are single degree of freedom model and only the tooth stiffness included as the potential energy storing element in the system rest of the elements flexibilities are neglected. (3) Gear dynamic model: include the flexibilities of other elements along with tooth compliance. (4) Geared rotor dynamics model: torsional vibration of system is included in the model along with whirl, gyroscopic effect, and time varying mesh stiffness.

Sometimes, it is very difficult to separate all these from each other. When a gear tooth is in contact that consists at two main components of the load in respect of transmitted power: 1. Static component, and 2. Dynamic component (due to fluctuating conditions). So, many researchers provide the dynamic factor, which is defined as the ratio of static load, and dynamic load[95,96].

$$Dynamic\ factor = \frac{Static\ load}{Dynamic\ load} \tag{1}$$

The concept of the speed is introduced at a later stage and the modified dynamic factor as stated:

$$(Dynamic\ factor)_{modifeid} = \frac{600}{(600 + pitch\ line\ velocity\ (in\ feet\ per\ minute))} \tag{2}$$

A report is published in 1931 by the ASME research committee on strength of gear teeth by Buckingham. The report suggested that “speed over 5000 fpm, the change in load carrying capacity of gear negligible, instead depends more on the effective mass, effective error”. Buckingham dynamic load factor is given as:

$$F_d = F_t + \frac{21\ v(b \times C + F_t)}{21\ v + \sqrt{b \times C + F_t}} \tag{3}$$

Where, Fd is the total load on gear including load due to dynamic action, Ft is the maximum tangential load, C is the load stress factor $\left(C = \frac{k \times e}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)}\right)$ in N/mm, k is a factor according to the pressure angle of the gear $\left(k = \begin{cases} 0.107 & for\ 14.4^{\circ}\ involute\ full\ depth \\ 0.111 & for\ 20^{\circ}\ involute\ full\ depth \\ 0.115 & for\ 20^{\circ}\ involute\ stub \end{cases}\right)$, Ep & Eg are modulus of elasticity of pinion and gear material, e is the sum of errors between two meshing teeth (mm). The gear tooth is designed by considering the gear as a cantilever beam. The Lewis formula is used to express the load[97]. The design of the gear tooth influences the dynamic characteristics. The optimisation of the design can bring the considerable reduction in the gear vibration amplitudes[98]. The influence of different parameters like load, speed, design, friction, roughness are studied to see the change in dynamic characterisation [80,99–109]. The transmission error is the one of the criteria for the change in the condition of the gear, the static and dynamic transmission error are effected by the type of fault [96,110–112]. The transmission error is defined as ” the difference between the angle rotated by the pinion and the correspondence angle rotated by gear” [113].

The Table 1 and Table 2 showing the different dynamic models considering the effect of only interaction between the gear teeth and including the effect of the other system elements like motor, shaft, bearings and the load respectively.

Table 1 Gear dynamic models [114–117]

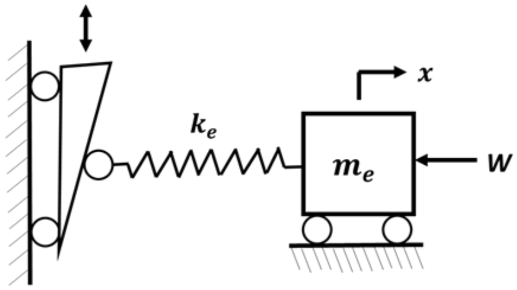
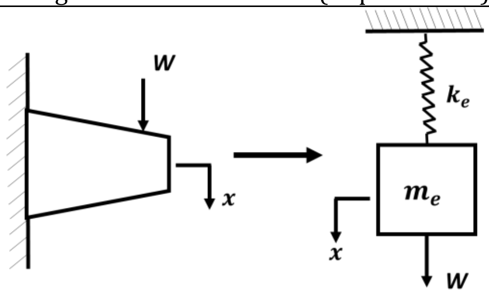
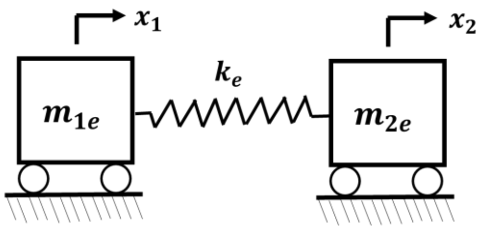
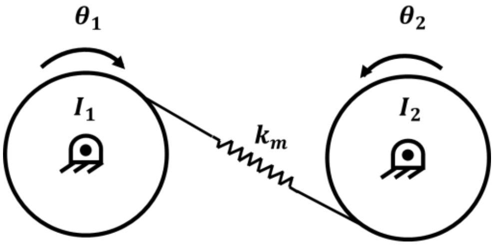
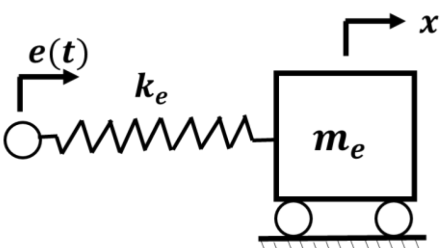
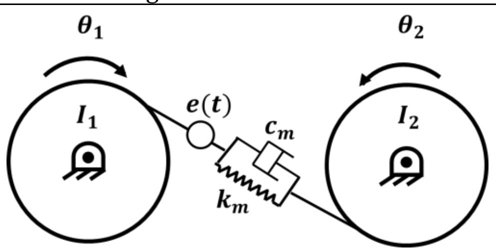
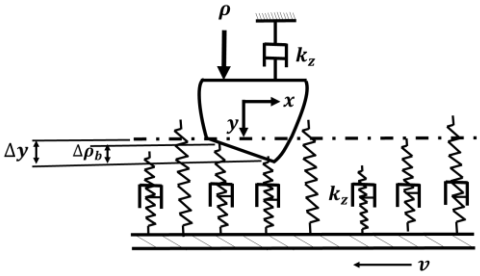
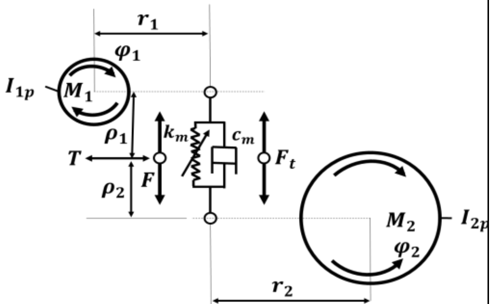
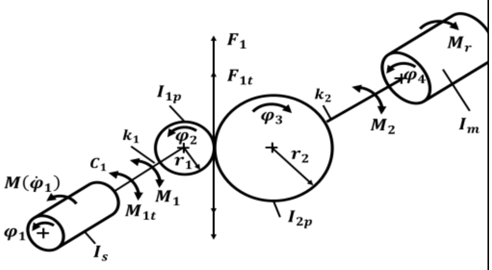
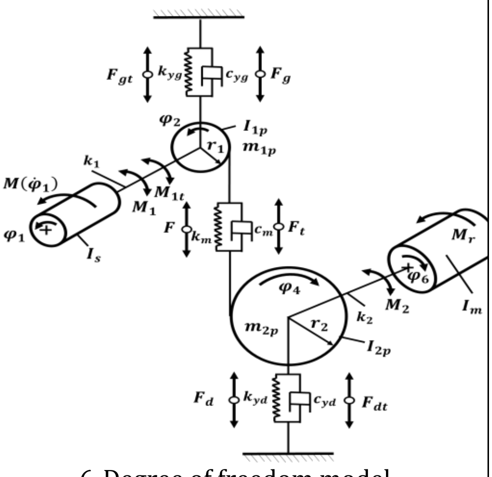
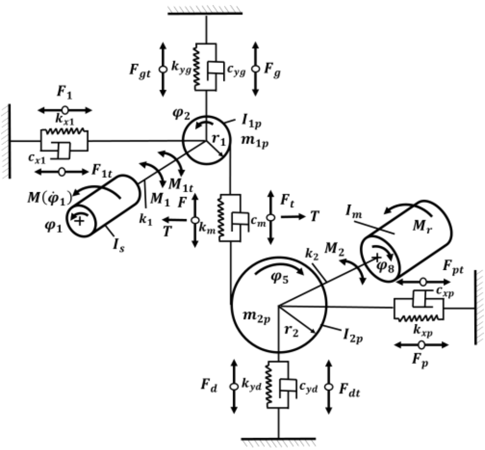
(a)	 1-Degree of freedom model (Tuplin model)	$m_e\ddot{x} + k_e(t)x = W$ Where, $k_e(t)$ is equivalent time dependent mess stiffness of contacting teeth, m_e equivalent mass, and W excitation force
(b)	 1-Degree of freedom model	$m_e\ddot{x} + k_e x = W$ Where, k_e is equivalent mess stiffness of contacting teeth, m_e equivalent mass, and W excitation force.
(c)	 2-Degree of freedom model	$m_{1e}\ddot{x}_1 = k_e(x_2 - x_1)$ $m_{2e}\ddot{x}_2 = -k_e(x_2 - x_1)$ Where, k_e is equivalent mess stiffness of contacting teeth, m_{1e} and m_{2e} equivalent masses of the gear wheels
(d)	 2-Degree of freedom model	$I_1\ddot{\theta}_1 = k_m(\theta_2 - \theta_1)$ $I_2\ddot{\theta}_2 = -k_m(\theta_2 - \theta_1)$ Where, k_m is equivalent mess stiffness of contacting teeth, I_1 and I_2 rotational moment of inertia
(e)	 1-Degree of freedom model	$m_e\ddot{x} = k_e[x - e(t)]$ Where, k_e is equivalent mess stiffness of contacting teeth, m_e equivalent mass, and *e(t)* is error function (it is also work as a excitation function also)
(f)	 2-Degree of freedom model	$[I]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(t)\}$ $[I] = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$, $q = r_1\theta_1 + r_2\theta_2 + e(t)$ $K = k_m(t) = k_o + \sum_{i=1}^{n} k_k \cos(i\omega_m t +)$

Table 2 Geared rotor Dynamic model [114–116,118]

(a)	 1-Degree of freedom model	It is a two-parameter model: stiffness and damping. The inertia of rotating wheels reduced to one mass. Motion of mass is equivalent to the relative motion of the wheels. The motion of springs with velocity v (m/s) is equivalent to the pitch-line velocity of the wheels. The stiffness of the system is represented with the different length springs. The motion of the mass has no influence on the change in velocity of the actual gearbox.
(b)	 2-Degree of freedom model	$I_{1p}\ddot{\varphi}_1 = M_1 \pm r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_2 = -M_2 + r_2(F + F_t) - M_{ft2}$ Where, M_{ft1} and M_{ft2} are friction forces $M_1 = k_m(\varphi_1 - \varphi_2)$, $M_2 = k_m(\varphi_2 - \varphi_1)$ $F = k_f(r_1\varphi_2 - r_2\varphi_3)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_3)$
(c)	 4- Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F_1 + F_{1t}) + M_{ft1}$ $I_{2p}\ddot{\varphi}_3 = -M_2 + r_2(F_1 + F_{1t}) - M_{ft2}, I_m\ddot{\varphi}_4 = M_2 - M_r$ Where, M_{ft1} and M_{ft2} are friction forces Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2), M_2 = k_2(\varphi_3 - \varphi_4)$ $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2), F_{1t} = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_3)$ $F_1 = k_f(r_1\varphi_2 - r_2\varphi_3)$
(e)	 6-Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_4 = -M_2 + r_2(F + F_t) - M_{ft2},\ I_m\ddot{\varphi}_6 = M_2 - M_r$ $m_{1p}\ddot{y}_{1p} = F + F_t - F_g - F_{gt}$, $m_{2p}\ddot{y}_{2p} = F + F_t - F_d - F_{dt}$ Where, M_{ft1} and M_{ft2} are friction forces, y describes vertical motion Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2)$, $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2)$ $M_2 = k_2(\varphi_4 - \varphi_6)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_4)$ $F = k_f(r_1\varphi_2 - r_2\varphi_4)$, $F_g = k_{yg}y_{1p}$ $F_d = k_{yd}y_{2p}$, $F_{gt} = C_{yg}\dot{y}_{1p}\ F_{dt} = C_{yd}\dot{y}_{2p}$

(e)	 8-Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_5 = -M_2 + r_2(F + F_t) - M_{ft2}$, $I_m\ddot{\varphi}_8 = M_2 - M_r$ $m_{1p}\ddot{y}_{1p} = F + F_t - F_g - F_{gt}$, $m_{1p}\ddot{x}_{1p} = T - F_1 - F_{1t}$ $m_{2p}\ddot{y}_{2p} = F + F_t - F_d - F_{dt}$, $m_{2p}\ddot{x}_{2p} = T - F_p - F_{pt}$ Where, M_{ft1} and M_{ft2} are friction forces, x and y describe horizontal and vertical motions Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2)$, $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2)$ $M_2 = k_2(\varphi_5 - \varphi_8)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_5)$ $F = k_f(r_1\varphi_2 - r_2\varphi_4)$, $F_g = k_{yg}y_{1p}$ $F_d = k_{yd}y_{2p}$, $F_{gt} = C_{yg}\dot{y}_{1p}$ $F_{dt} = C_{yd}\dot{y}_{2p}$, $F_1 = k_{x1}x_{1p}$ $F_p = k_{xp}x_{2p}$, $F_{1t} = C_{x1}\dot{x}_{1p}$ $F_{pt} = C_{xp}\dot{x}_{2p}$

2.1.2 Contact stiffness estimation in gears

Contact stiffness is described as the resultant of the stiffness of meshing teeth of the gears. Mesh stiffness is the function of number of teeth in contact, tooth geometry, application and type of load, material properties of the gear, and geometrical modification due to profile errors/ faults in gear tooth[119,120]. In few literature the back side contact gear mesh stiffness and torsional stiffness is determined[121,122]. In recent studies the mesh stiffness is calculated by incorporating the stiffness of the lubricant between the meshing teeth[123]. The normal and tangential component of the meshing stiffness are calculated separately and then added in two combination: (a) In series, and (b) In parallel. The hypothesis of equal shear stress on laminar element is applied and the effect of parameters (operating and geometrical) on lubricant film is considered like operating force, speed, and tooth geometry. Then the equivalent stiffness is the final output[124,125]:

$$K_e = K_t + K_l \quad (4)$$
$$K_t = K_m \quad (5)$$
$$K_l = K_n + K_{tangential} \quad (6)$$
$$K_{n/tangential} = \frac{\Delta F_{n/tangential}(t)}{\Delta x_{n/tangential}} \quad (7)$$

Mesh stiffness of gear tooth can be estimated by the following ways[126]:

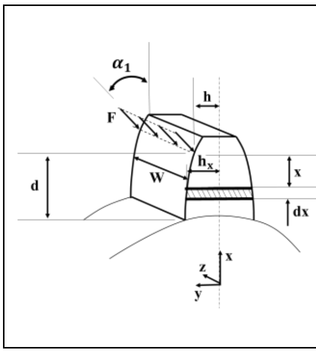
1. *Square waveform method*

In this method, for a constantly running gear pair, the mesh stiffness is a periodic function, and a square waveform is used to approximate the mesh stiffness of the gear teeth. The period, and the duration of the waveform equal to the period of mesh period of teeth, and time duration of one revolution. The limitation of the using square waveform method is that change in contact position is ignored, there is not any standard formulation for the magnitude approximation, the reduction in the mesh stiffness is also subjective to the knowledge of the fault, and its severity.

2. *Potential energy method*

The gear tooth is modelled as a non-uniform cantilever beam. The mesh stiffness of the gear is the algebraic addition of the bending stiffness, shear stiffness, compressive stiffness, Hertzian stiffness. The effect of tooth fillet also considered for estimation the gear stiffness. The tooth fillet effect equation is quite useful for the healthy teeth, but for faults like crack, wear, and pitting, the equation is not valid. It's quite challenging to get the exact contact location during the rotation in the gear[123]. The problem is solved by driving all the stiffnesses bending, shear, compressive, Hertzian as a function of the rotation angle of the gear.

Table 3 Potential energy stored in meshing gear teeth[127–135]

 	$\frac{1}{K_b} = \int_0^d \frac{(x.\cos(\alpha_1) - h.\sin(\alpha_1))^2}{E.I_x} dx$ $\frac{1}{K_s} = \int_0^d \frac{1.2\cos^2(\alpha_1)}{G.A_x} dx$ $\frac{1}{K_a} = \int_0^d \frac{\sin^2(\alpha_1)}{E.A_x} dx$ $K_h = \frac{\pi EW}{4(1 - v^2)}$

Meshing stiffness is given by [127,136]

$$K_m = \sum_{i=1}^{n} \frac{1}{\left(\frac{1}{K_{1bi}}+\frac{1}{K_{1si}}+\frac{1}{K_{1ai}}+\frac{1}{K_{1hi}}+\frac{1}{K_{2bi}}+\frac{1}{K_{2si}}+\frac{1}{K_{2ai}}+\frac{1}{K_{2hi}}\right)} \quad (8)$$

The subscript 1 and 2 are denoting the driving and driven gear. The above equation can be directly used to evaluate the meshing stiffness without the root fillet effect. To evaluate the stiffness by this method requires better understanding of the contact physics.

3. Finite element method

In the finite element modelling (FEM) evaluation of gear mesh stiffness is not required. The evaluation of stiffness is required in dynamic model. The approaches used for evaluation of gear mesh stiffness are average slope (static analysis), local slope (dynamic analysis). The models are developed liner to save the computational time and cost. The mesh stiffness calculation through this method is a function of mesh density, type of finite element, and contact physics. In most of the finite element model the gear rotor is modelled as simply supported beam element [137].

4. Experimental method

It is quite difficult to measure the stiffness of gear in all direction. For, measuring the mesh stiffness experimentally; the following factors are mandatory: stress intensity on the tooth surface, and elastic deformation. The dynamic deformation of the gear surface is mapped with the help speckle photography. The experimental methods are tested under dynamic and the static condition. The agreement between model based and the experiment are not confirmed in any literature.

2.1.3 Damping estimation in the gears

The damping is a highly non-linear phenomenon and it depends upon material properties, operating, and geometrical conditions[77,138]. In most gear dynamic studies, the damping is incorporated as a fixed value depending upon the material properties. In most cases, the value for the damping coefficient (ξ=0.1). The damping coefficient proportional to the total mesh stiffness ($c = \mu K_m$) is also reported in some literatures [129]. The μ is a scale constant having units of time.in few literatures for the damping ratio estimation the equation is further modified and expressed more explicitly as: $\xi = \frac{C_m}{2\sqrt{K_m m_e}}$, the formulation consist the effect of equivalent inertia, average mesh stiffness and viscous damping. In gear, the damping consists of the following sources: surrounding elements, hysteresis of teeth, and fluid damping (squeeze and shear). First two are estimated with the help of the load support by the element and gear teeth, and the third one is evaluated from the Reynold's equation. The teeth damping is evaluated by hysteresis of teeth. The damping is evaluated as the structural damping or Rayleigh damping. The gear teeth are worked under the elastohydrodynamic lubrication condition in active teeth. The Reynold equation is solved for the both loaded and no-load condition. The equation is solved by neglecting the axial direction losses, and if the lubricant incompressible and viscous fluid. The gear flanks are in line contact due to non-conformal contacts. So, Hertzian theory of contact plays important role in estimation of the pressure[139]. During tooth separation the no-load condition is considered. During this period the deformation due to Hertzian effect is no longer applicable. Without considering the axial flow, the Reynold's equation[140] is expressed as [141]:

$$\frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 12\,\mu \bar{u} \frac{\partial h}{\partial y} + 12\,\mu \frac{\partial h}{\partial t} \quad (9)$$

In few literatures works, the combined stiffness and damping is derived from the lubricant film and the teeth. The dynamic pressure and film thickness are evaluated. The Reynold equation help in finding the dynamic pressure (normal and shear), the film thickness in normal compression rate, and the tangential direction speed; damping can be evaluated with the expression given as the ratio of dynamic force (normal or tangential direction) and velocity (normal or tangential direction). The damping is given as[125]:

$$C_{n/\tau} = \frac{\Delta F_{n/\tau}}{\Delta \dot{u}_{n/\tau}} \quad (10)$$

The subscript n and τ are denoting the normal and tangential direction. In some literature the damping is calculated by the trace method (force- displacement curve). By this method the equivalent viscous damping is evaluated form the area enclosed by one complete oscillation.

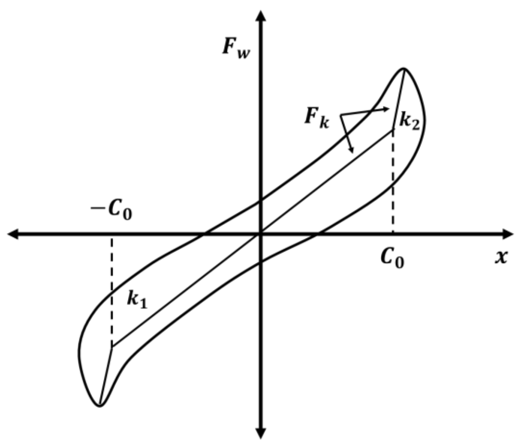


Fig. 1 Nonlinear teeth contact force and trace force function (hysteresis loop)[142]

Conclusions

This paper reviewed the aspects to model to mimic the system condition based on many assumptions to simply the modelling and to reduce computational cost.

a. The gear mesh stiffness evaluation equations are only valid for the narrow approach like single tooth contact only. The equation should be broadened to incorporate the effect of the multi-tooth contact.
b. The gear mesh stiffness calculated with the lubricant stiffness are lacks the rheological effect of the lubricant under the single and multi-tooth contact.
c. Most of the dynamic calculation are done by considering the gear mesh damping constant, equivalent to the material damping of the gear. The effect of non-linearity of the damping needs some investigation.
d. The effect of the lubricant and its rheology needs to incorporate in meshing stiffness evaluation.
e. The hybrid faults on multiple teeth needs some investigation.
f. The effect of the faults of the supporting element in the system like bearing fault, shaft faults need some investigation.
g. There limited models are available on basis of viscoelastic fracture mechanics. Most of the researcher assumed gear body as rigid body, this assumption leads some amount of error under high speed or high frequency excitation.
h. The model for time varying excitation sources deserves some level of investigation.
i. A model with optimum degree of freedom needs to be defined with minimum error. The high degree of freedom leads to high computational resources and low degree of freedom model may lead large error between model and real system.

Nomenclature

C	Load stress factor
F_d	Total dynamic load on gear tooth
F_t	Tangential load on gear tooth
v	Spring motion velocity (m/s)
e	Meshing error between gear teeth
k_e	Time dependent mess stiffness
m_e	Equivalent mass
W	Excitation force
x, y, z	Coordinate along rolling, squeezing and axial/film direction
X, Y, Z	Non-dimensional coordinate system
$e(t)$	Error function
k_m	Gear mesh stiffness
c_m	Gear mesh damping
M	Moment
F	Force
r	Radius of gear
K_a	Axial or compressive stiffness (N/m)
K_b	Bending stiffness (N/m)
K_s	Shear stiffness (N/m)
K_h	Hertzian stiffness (N/m)
K_m	Equivalent mesh stiffness (N/m)
E	Modulus of elasticity (GPa)
v	Poisson's ratio
G	Modulus of rigidity (GPa)
I	Moment of inertia (kg. m2)

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