

Research Article

# Minimizing the Cost in Balanced Transportation Problem by using Different Means

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## Abstract

In science, business and Mathematics matrices are important tool which is used in almost all branches of mathematics. Because the data is distributed in rows and columns by that arrangement, we can easily find out the position of unit. So linear programming uses transportation problem like matrices and transportation problem is very important tool for business in every aspect of life. Transportation problem plays vital role in business economy of every country to overcome the costs of goods. To solve balanced transportation problem there are many methods like LCM, NWCM and VAM are used to minimize the cost and maximize the profit. To overcome maximum transportation cost we use different means for balanced transportation problem to minimize the total cost of transportation problem that is initial basic feasible solution. The proposed algorithm gives better result than existing methods which are available for balanced transportation problems.

**Keywords:** Balanced transportation problem, Different means, Vogel approximation method, Arithmetic Mean, Geometric Mean, Harmonic Mean and Contra Harmonic Mean.

## Introduction

Linear programming deals with the problems of optimization and finding the maximum or minimum value of linear function under certain condition. Linear programming optimization problems are solved by operation research technique. Linear programming problem also devoted with transportation class. Operation research deals with the balanced and unbalanced transportation problems to get initial basic feasible solution and minimized cost as well as optimal solution. In operation research models some transportation problems are used which concerned with maximization or minimization of cost to reach destination. When each demand is supplies to source supply and demand equal to reach destination such type of transportation problems is called as balanced and if supply does not equal to demand then these problems are called as unbalanced transportation problem. Transportation problem algorithm initially developed by Hitchcock during 2nd world War. Transportation problem plays vital role in supply chain problems as well as in optimization technique and also in Mathematics, Engineering, Business and economics.

Transportation problems are application of linear programming and the application of transportation problems are concerned with factories, manufacturer companies which sold goods in market to maximize the profit and also to give good service to their customer in least cost.

There are so many techniques of optimization for balanced and unbalanced transportation problems are used get initial basic feasible solution then optimal solution but some were found optimal solution and other found basic feasible solution like such as Vogel Approximation method (VAM), Least Corner Method (LCM), North West Corner Method (NWCM), Matrix Minima Method, Invariant Vogel Approximation Method (IVAM) and Modified Vogel Method (MVAM) and for Optimal Solution method or near to optimal Modi Method, Zero Point Method and Harmonic Mean Technique Method etc. Our focus will be on balanced transportation problems to find least cost and optimal solution or near to optimal solution by using analysis of different means technique.

## General Transportation Problem Table

There are  $n$  sources and  $m$  destination, each represent by  $c$  "node". The relationship of launching point and terminal point represent the route between the launching and the terminal points. Arcs  $(i,j)$  joining launching  $i$  to terminal  $j$  carries two pieces of information. Transportation cost per unit  $C_{ij}$  and the amount shipped  $X_{ij}$ , where  $a_i$  be number of supply at

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source  $i$  and  $b_j$  be number of demand at destination  $j$ . The objective of the model is to determine the unknowns  $X_{ij}$  that will minimize the total transportation cost while satisfying all the supply and demand restriction.

$$\text{Min}Z = \sum C_{ij}X_{ij}$$

$$\text{s.t: } x_{ij} = S_i \quad i=1,2,3,4,\dots,m, \quad x_{ij} = D_j \quad j=1,2,3,4,\dots,n \text{ and } x_{ij} \geq 0, \forall i,j$$

Where:  $a_i$  = Supply at origin  $i$  and  $b_j$  = Demand at destination  $j$

$x_{ij}$  = Number of units shipped from origin to terminal point

Linear programming model of transportation problem

Origin(i)	Destination(j)				Supply( $a_i$ )
	D1	D2	....	Dn	
S1	C11	c12	....	D1n	a1
S2	C21	c22	....	D2n	a2
....	....	....	....	....	....
Sn	cm1	cm2	....	Dmn	am
Demand( $b_j$ )	b1	b2	....	bn	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

**Feasible Solution**

A non-negative allocation  $X_{ij}$  so which satisfy the row and column restriction is known as Basic feasible solution.

**Basic Feasible Solution**

A feasible solution to  $m$  origin and  $n$  destination problem is said to be basic feasible solution. If the number of positive allocations is  $(m+n-1)$

If the number of allocations in a basic feasible solution is less than  $(m+n-1)$  it is called degenerated basic feasible solution otherwise non-degenerated.

**Optimal Solution**

A feasible solution not necessarily is to be optimal if it minimizes the total transportation cost.

**Algorithms of initial basic feasible solution methods**

**Vogel Approximation Method**

Vogel approximation method is a trial-and-error and usually provides initially a better starting solution than other existing methods. Application of VAM to a give does not guarantee that an optimal solution will result. However, its result is invariably obtained with comparatively little effort.

**Step:1.** Determine the difference between the lowest two cells in all rows and columns.

**Step:2.** Identify the row or column with the largest difference ties may be broken arbitrarily.

**Step:3.** Allocate as much as possible to the lowest-cost cell in the row or column with the highest differences. If two or more differences are equal, allocate as much as possible to the lowest-cost cell in these rows or columns.

**Step:4.** Stop the process if all row and go to next step column requirement are met, if not

**Step:5.** Recalculate the differences between the two lowest cells remaining in all rows and columns. Any row and column with zero supply demand should not be used in calculating further differences than go to step-2

The VAM usually produces an optimal or near-optimal starting solution. One study found that VAM yields an optimum solution in 80% of the sample problems tested.

**North-West Corner Method (NWCM)**

Algorithm

**Step 1.** Select the North-West (upper left-hand) corner cell of the transportation table and allocate units according to the supply and demand.

**Step 2.** If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

**Step 3.** If the supply for the first row is exhausted, then move down to the first cell in the second row.

**Step 4.** Continue the process until all supply and demand values are exhausted.

**Least Cost Method (LCM)**

Algorithm

**Step 1.** First examine the cost matrix and choose the cell with minimum cost and then allocate there as much as possible. If such a cell is not unique, select arbitrary any one of these cells.

**Step 2.** Cross out the satisfied row or a column. If either a column or a row is satisfied simultaneously, only one may be crossed out.

**Step 3.** Write the reduced transportation table and repeat the process from step 1 to step 2, until one row or one column is left out.

**Optimal Method**

**Modified Distribution Modi Method**

This method always gives the total minimum transportation cost to transport the goods from sources to the destinations.

**Algorithm**

**Step1.** If the problem is unbalanced, balance it. Setup the transportation tableau

**sStep2.** Find a basic feasible solution.

**Step3.** Set  $u_1 = 0$  and determine  $u_i$ 's and  $v_j$ 's such that  $u_i + v_j = c_{ij}$  for all basic variables.

**Step4.** If the reduced cost  $0 c_{ij} - u_i - v_j \geq 0$  for all non-basic variables (minimization problem), then the current BFS is optimal. Stop! Else, enter variable with most negative reduced cost and find leaving variable by looping.

**Step5.** Using the new BFS, repeat steps 3 and 4.

**Minimizing the Cost in Balanced Transportation Problem by using Different Means**

**Proposed Algorithm**

**Step1.** Find the average (Geometric mean, Arithmetic mean, Contraharmonic mean, Heronian mean) through each row.

**Step2.** Find the average (Geometric mean, Arithmetic mean, Contra harmonic mean, Heronian mean) through each column.

**Step3.** Identify the highest penalty in row or column

**Step4.** Then supply the cost to least unit cell.

**Step5.** Repeat step above steps to supply each cost to unit cell.

**Numerical Example (1)**

Origin	Destinations				Supply
	D1	D2	D3	D4	
S1	6	3	8	7	110
S2	8	5	2	4	60
S3	4	9	8	4	54
S4	7	8	5	6	30
Demand	20	70	78	86	$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

First, we find the row average and column average which are shown in table

Penalty Distribution Table

Source	Destination				Supply	A.M	Geometric mean	Contra harmonic mean	Heronian Mean
	S1	S2	S3	S4		$\bar{x} = \sum xi/n$	G.M = $(a.b)^{1/n}$	CHM = $a^2 + b^2/a+b$	HM = $1/3(2A M+GM)$
D1	6	3	8	7	110	6	5.6	6.6	5.9
D2	8	5	2	4	60	4.8	4.2	5.7	4.6
D3	4	9	8	4	54	6.3	5.8	7.1	6.1
D4	7	8	5	6	30	6.5	6.4	6.7	6.5
Demand	20	70	78	86	254				
$\bar{y} = \sum yi$	6.3	6.3	5.8	5.3					
G.M = $(a.b)^{1/n}$	6.1	5.7	5	5.1					
CHM = $a^2 + b^2/a+b$	6.6	7.2	6.8	5.6					
HM = $1/3(2AM+GM)$	6.2	6.1	5.5	5.2					

**Solution of Balanced Transportation Problem by Arithmetic Mean**

Origin	Destinations				Supply	Arithmetic Mean
Source	D1	D2	D3	D4		
S1	6	3	8	7	110	6
S2	8	5	2	4	60	4.8
S3	4	9	8	4	54	6.3
S4	7	8	5	6	30	6.5
Demand	20	70	78	86		

At first we find the row average and column average which are shown in Table

Source	Destinations				Supply	Arithmetic Mean
	D1	D2	D3	D4		
S1	6	3	8	7	110	6
S2	8	5	2	4	60	4.8

S <sub>3</sub>	4	9	8	4	54	6.3
S <sub>4</sub>	7	8	5J <sup>30</sup>	6	30	6.5
Demand	20	70	78	86		
Arithmetic mean	6.3	6.3	5.8	5.3		

At second supply to lowest unit cost of maximum penalty of row or in column

Source	Destinations				Supply	Arithmetic mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	6
S <sub>2</sub>	8	5	2	4	60	4.8
S <sub>3</sub>	4J <sup>20</sup>	9	8	4	54	6.3
Demand	20	70	48	86		
Arithmetic mean	6	5.7	6	5		

	Destinations			Supply	Arithmetic Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	3	8	7	110	6
S <sub>2</sub>	5	2	4	60	3.7
S <sub>3</sub>	9	8	4J <sup>34</sup>	34	7
Demand	70	48	86		
Arithmetic mean	5.7	6	5		

Source	Destinations			Supply	Arithmetic Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	3J <sup>70</sup>	8	7	110	6
S <sub>2</sub>	5	2	4	60	3.7
Demand	70	48	52		
Arithmetic mean	4	5	5.5		

Origin	Destinations		Supply	Arithmetic Mean
	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	8	7J <sup>40</sup>	40	7.5
S <sub>2</sub>	2J <sup>48</sup>	4J <sup>12</sup>	60	3
Demand	48	52		
Arithmetic Mean	5	5.5		

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	6	3J <sup>70</sup>	8	7J <sup>40</sup>	110
S <sub>2</sub>	8	5	2J <sup>48</sup>	4J <sup>12</sup>	60
S <sub>3</sub>	4J <sup>20</sup>	9	8	4J <sup>34</sup>	54
S <sub>4</sub>	7	8	5J <sup>30</sup>	6	30
Demand	20	70	78	86	

$$\begin{aligned}
 &C_{12}X_{12} + C_{14}X_{14} + C_{23}X_{23} + C_{24}X_{24} + C_{31}X_{31} + C_{24}X_{24} + C_{33}X_{33} \\
 &3 \times 70 + 7 \times 40 + 2 \times 48 + 4 \times 12 + 4 \times 20 + 4 \times 34 + 5 \times 30 \\
 &210 + 280 + 96 + 48 + 80 + 136 + 150 \\
 &1000
 \end{aligned}$$

**Solution of Balanced Transportation Problem by Geometric Mean**

At first we find the row average and column average which are shown in Table

Origin	Destinations				Supply	Geometric Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	6
S <sub>2</sub>	8	5	2	4	60	4.8
S <sub>3</sub>	4	9	8	4	54	6.3
S <sub>4</sub>	7	8	5	6	30	6.5
Demand	20	70	78	86		
Geometric mean	6.3	6.3	5.8	5.3		

At second supply to lowest unit cost of Maximum penalty of row or in column

	Destinations				Supply	Geometric Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	6
S <sub>2</sub>	8	5	2	4	60	4.8
S <sub>3</sub>	4	9	8	4	54	6.3

S <sub>4</sub>	7	8	5J <sup>30</sup>	6	30	6.5
Demand	20	70	78	86		
Geometric mean	6.3	6.3	5.8	5.3		

	Destinations				Supply	Geometric mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	6
S <sub>2</sub>	8	5	2	4	60	4.8
S <sub>3</sub>	4J <sup>20</sup>	9	8	4	54	6.3
Demand	20	70	48	86		
Geometric mean	6	5.7	6	5		

	Destinations			Supply	Geometric mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	3	8	7	110	6
S <sub>2</sub>	5	2	4	60	3.7
S <sub>3</sub>	9	8	4J <sup>34</sup>	34	7
Demand	70	48	86		
Geometric mean	5.7	6	5		

	Destinations			Supply	Geometric mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	3J <sup>70</sup>	8	7	110	6
S <sub>2</sub>	5	2	4	60	3.7
Demand	70	48	52		
Geometric mean	4	5	5.5		

Source	Destinations		Supply	Geometric mean
	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	8	7J <sup>40</sup>	40	7.5
S <sub>2</sub>	2J <sup>48</sup>	4J <sup>12</sup>	60	3
Demand	48	52		
Geometric mean	5	5.5		

Origin	Destinations				Supply
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
S <sub>1</sub>	6	3J <sup>70</sup>	8	7J <sup>40</sup>	110
S <sub>2</sub>	8	5	2J <sup>48</sup>	4J <sup>12</sup>	60
S <sub>3</sub>	4J <sup>20</sup>	9	8	4J <sup>34</sup>	54
S <sub>4</sub>	7	8	5J <sup>30</sup>	6	30
Demand	20	70	78	86	

$$\begin{aligned}
 &C_{12}X_{12} + C_{14}X_{14} + C_{23}X_{23} + C_{24}X_{24} + C_{31}X_{31} + C_{24}X_{24} + C_{33}X_{33} \\
 &3 \times 70 + 7 \times 40 + 2 \times 48 + 4 \times 12 + 4 \times 20 + 4 \times 34 + 5 \times 30 \\
 &210 + 280 + 96 + 48 + 80 + 136 + 150 \\
 &1000
 \end{aligned}$$

**Solution of Balanced Transportation Problem By Contra Harmonic Mean**

At first we find the row average and column average which are shown in Table

Source	Destinations				Supply	Contra Harmonic Mean C.H.M
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	6.6
S <sub>2</sub>	8	5	2	4	60	5.7
S <sub>3</sub>	4	9	8	4	54	7.1
S <sub>4</sub>	7	8	5	6	30	6.7
Demand	20	70	78	86		
C.H.M	6.6	7.2	6.8	5.6		

At second supply to lowest unit cost of maximum penalty of row or in column

Source	Destinations				Supply	C.H.M
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3J <sup>70</sup>	8	7	110	6.6
S <sub>2</sub>	8	5	2	4	60	5.7
S <sub>3</sub>	4	9	8	4	54	7.1
S <sub>4</sub>	7	8	5	6	30	6.7
Demand	20	70	78	86		
C.H.M	6.6	7.2	6.8	5.6		

Source	Destinations			Supply	C.H.M
	D <sub>1</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6J <sup>20</sup>	8	7	40	7.1
S <sub>2</sub>	8	2	4	60	6
S <sub>3</sub>	4	8	4	54	6
S <sub>4</sub>	7	5	6	30	6.1
Demand	20	78	86		
C.H.M	6.6	6.8	5.6		

	Destinations			Supply	C.H.M
	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	8	7	20		7.5
S <sub>2</sub>	2J <sup>60</sup>	4	60		3.3
S <sub>3</sub>	8	4	54		6.7
S <sub>4</sub>	5	6	30		5.5
Demand	78	86			
C.H.M	6.8	5.6			

	Destinations		Supply	C.H.M
	D <sub>1</sub>	D <sub>2</sub>		
S <sub>1</sub>	8	7J <sup>20</sup>	20	7.5
S <sub>2</sub>	8	4	54	6.7
S <sub>3</sub>	5	6	30	5.5
Demand	18	86		
C.H.M	7.3	5.9		

	Destinations		Supply	C.H.M
	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	8	4J <sup>54</sup>	54	6.7
S <sub>2</sub>	5J <sup>18</sup>	6J <sup>12</sup>	30	5.5
Demand	18	66		
C.H.M	6.8	5.2		

Origin	Destinations				Supply
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
S <sub>1</sub>	6J <sup>20</sup>	3J <sup>70</sup>	8	7J <sup>20</sup>	110
S <sub>2</sub>	8	5	2J <sup>60</sup>	4	60
S <sub>3</sub>	4	9	8	4J <sup>54</sup>	54
S <sub>4</sub>	7	8	5J <sup>18</sup>	6J <sup>12</sup>	30
Demand	20	70	78	86	

$$\begin{aligned}
 &C_{11}X_{11} + C_{12}X_{12} + C_{13}X_{13} + C_{23}X_{23} + C_{34}X_{34} + C_{43}X_{43} + C_{44}X_{44} \\
 &6 \times 20 + 3 \times 70 + 7 \times 20 + 2 \times 60 + 4 \times 54 + 5 \times 18 + 6 \times 12 \\
 &120 + 210 + 140 + 120 + 216 + 90 + 72 \\
 &968
 \end{aligned}$$

**Solution of Balanced Transportation Problem by Heronian Mean**

At first we find the row average and column average which are shown in Table

	Destinations				Supply	Heronian Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	5.9
S <sub>2</sub>	8	5	2	4	60	4.6
S <sub>3</sub>	4	9	8	4	54	6.1
S <sub>4</sub>	7	8	5	6	30	6.5
Demand	20	70	78	86		
Heronian Mean	6.2	6.1	5.5	5.2		

Origin	Destinations				Supply	Heronian Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	6	3	8	7	110	5.9
S <sub>2</sub>	8	5	2	4	60	4.6
S <sub>3</sub>	4J <sup>20</sup>	9	8	4	54	6.1
S <sub>4</sub>	7	8	5	6	30	6.5
Demand	20	70	78	86		
Heronian Mean	6.2	6.1	5.5	5.2		

	Destinations			Supply	Heronian Mean
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>		
S <sub>1</sub>	3	8	7	110	5.8
S <sub>2</sub>	5	2	4	60	3.6
S <sub>3</sub>	8	5J <sup>30</sup>	6	30	6.3
Demand	70	78	52		
Heronian Mean	5.2	4.8	5.6		

	Destinations			Supply	Heronian Mean
	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	3	8	7	110	5.8
S <sub>2</sub>	5	2	4	60	3.6
S <sub>3</sub>	9	8	4J <sup>34</sup>	34	6.9
S <sub>4</sub>	8	5	6	30	6.3
Demand	70	78	86		
Heronian Mean	6.1	5.5			

	Destinations			Supply	Heronian Mean
	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	3J <sup>70</sup>	8	7	110	5.8
S <sub>2</sub>	5	2	4	60	3.6
Demand	70	48	52		
Heronian Mean	4	4.7	5.4		

	Destinations		Supply	Heronian Mean
	D <sub>3</sub>	D <sub>4</sub>		
S <sub>1</sub>	8	7J <sup>40</sup>	40	7.5
S <sub>2</sub>	2J <sup>48</sup>	4J <sup>12</sup>	60	2.9
Demand	48	52		
Heronian Mean	4.7	5.4		

Source	Destinations				Supply
	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
S <sub>1</sub>	6	3J <sup>70</sup>	8	7J <sup>40</sup>	110
S <sub>2</sub>	8	5	2J <sup>48</sup>	4J <sup>12</sup>	60
S <sub>3</sub>	4J <sup>20</sup>	9	8	4J <sup>34</sup>	54
S <sub>4</sub>	7	8	5J <sup>30</sup>	6	30
Demand	20	70	78	86	

$$\begin{aligned}
 &C_{12}X_{12} + C_{14}X_{14} + C_{23}X_{23} + C_{24}X_{24} + C_{31}X_{31} + C_{34}X_{34} + C_{43}X_{43} \\
 &3 \times 70 + 7 \times 40 + 2 \times 48 + 4 \times 12 + 4 \times 20 + 4 \times 34 + 5 \times 30 \\
 &210 + 280 + 96 + 48 + 80 + 136 + 150 \\
 &1000
 \end{aligned}$$

**Example (2)**

Origin	Destination				Supply
	D1	D2	D3	D4	
S1	20	30	50	17	7
S2	70	35	40	60	10
S3	90	12	60	25	18
Demand	5	8	7	15	Demand = Supply

**Example (3)**

Origin	Destination				Supply
	D1	D2	D3	D4	
S1	20	22	17	4	120
S2	24	37	9	7	70
S3	32	37	20	15	50
<b>Demand</b>	<b>60</b>	<b>40</b>	<b>30</b>	<b>110</b>	<b>Demand= Supply</b>

**Example (4)**

Origin	Destination			Supply
	D1	D2	D3	
S1	10	7	8	45
S2	15	12	9	15
S3	7	8	12	40
<b>Demand</b>	<b>25</b>	<b>55</b>	<b>20</b>	<b>Demand = Supply</b>

**Comparison of results**

Methods	Example 1	Example 2	Example 3	Example 4
<b>Proposed Method</b>				
Arithmetic Mean	1000	904	3460	750
Heronian Mean	1000	904	3460	750
Geometric Mean Mean	968	904	3460	750
Contra Harmonic Mean	968	904	3460	750
VAM	968	940	3520	750
North West Corner Method	1000	1085	3680	970
Least Cost method	1000	1085	3670	765
<b>Optimal Solution</b>	<b>968</b>	<b>904</b>	<b>3460</b>	<b>750</b>

**Conclusion**

In Paper Minimizing the cost in balanced transportation problem by different means I used VAM procedure with different means to minimized the cost which is called as least cost of shipping goods from company to terminal point. We can find the minimum cost of balanced transportation problem also one quality of this work is that we save our time by using proposed algorithm. From the comparison table, we can check out that the initial basic feasible solution and optimum solution or near to optimum solution obtained by the proposed method. The solution which is obtained by different means (arithmetic mean, geometric mean, Heronian mean and Contra Harmonic mean) is initial basic feasible solution that is also optimal solution. The proposed algorithm is more efficient than existing methods to get initial Basic feasible solution.

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