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Research Article

Mass Transfer Study of Free Convective Unsteady Flow through a Porous Medium with Induced Magnetic Field

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Abstract

A natural convective mass transfer transient flow of a viscous incompressible fluid past an infinite vertical plate bounded by a porous medium is investigated in presence of induced magnetic field. Mathematical model related to the problem is developed from the basis of studying magnetohydrodynamics(MHD) for the both lighter and heavier particles. Non-dimensional system of equations is obtained by the use of dimensionless quantities. The system of momentum, magnetic induction and concentration equations is solved numerically by finite difference technique. The chief physical interest as shear stress, current density and Sherwood number are also studied here. In order to discuss the physical aspects of the problem, the obtained numerical values of flow variables are plotted in graphs for different values of associated parameters. Last of all, some important findings of this observation are concluded in this study.

Keywords: *MHD, Mass Transfer, Free Convective, Unsteady Flow, Induced Magnetic Field*.

1. Introduction

The mass transfer processes play an important role in the production of materials in order to obtain the desired properties of a substance. The processes of separation in chemical engineering such as the drying of solid materials, distillation, extraction and absorption are all affected by the process of mass transfer. Chemical reactions including combustion processes are often decisively determined by the mass transfer. A free convective unsteady flow with mass transfer was studied (Callahan and Marner, 1976) along a semi-infinite plate. An investigation on free convective unsteady flow with mass transfer has been performed (Soundalgekar and Wavre, 1977) over an infinite vertical porous plate with constant suction. Mass transfer analysis on a free convective transient flow is completed (Soundalgekar and Ganesan, 1980) past a semi-infinite vertical plate.

The natural convective flows through a porous medium are of great interest in many industrial applications as to insulate the heated body to maintain its temperature. So it is necessary to study the flow through a porous medium. The steady free convective flow through a porous medium bounded by an infinite surface was investigated (Raptis *et al.*, 1981) by the use of the model of Yamamoto and Iwamura(1976). A natural convective flow about a vertical plate embedded in porous medium has been analyzed by Kim and Vafai (1989). Recently, a free convective unsteady flow has been studied (Magyari *et al*., 2004) in a porous medium.

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All the above problems are studied in the absence of induced magnetic field. However, the flow under the action of a strong magnetic field that induced another magnetic field has special interest in many industrial applications. Due to its practical importance, heat transfer study of free convective unsteady MHD flow past an infinite vertical plate has been completed (Haque *et al*., 2014) in presence of induced magnetic field. The induced magnetic field effect on the concentration level of fluid is not observed in the above problem. Hence, our main aim is to investigate a time dependent natural convective mass transfer MHD flow through a porous medium with the presence of induced magnetic field. These types of flow play a decisive role in many geophysical and astrophysical problems.

2. Mathematical Model of Flow

A free convective mass transfer unsteady flow of an electrically conducting incompressible viscous fluid past an electrically non conducting infinite vertical plate surrounded by a porous medium is considered here. The flow is assumed to be in the *x*-direction, which is chosen along the plate in upward direction and *y*-axis is normal to the plate. A strong magnetic field **B** is applied normal to the flow region that induced another magnetic field **H**. Initially, it is considered that the plate as well as the fluid particles are at rest at the same species concentration level $C = C_\infty$ at all points, where C_∞ is species concentration of uniform flow. The flow configuration with coordinate system is shown in Fig. 1.

Within the framework of the above stated assumptions, the equations relevant to the present problem under

Prandtl's boundary layer approximations are governed by the following system of coupled non-linear partial differential equations,

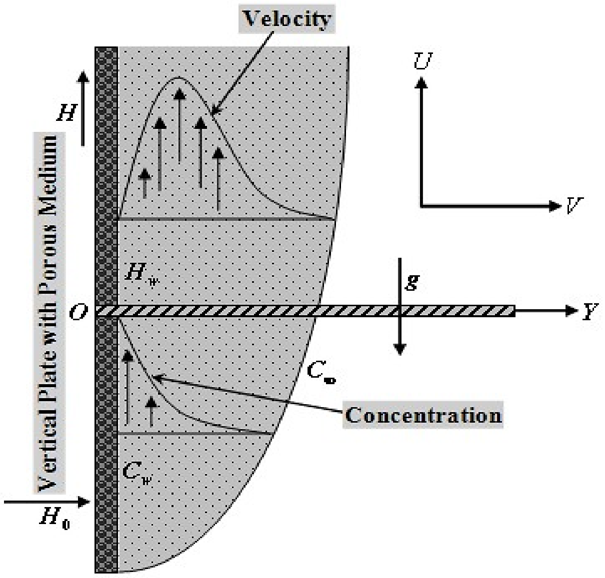

Fig. 1 Physical configuration with coordinate system

Continuity Equation

$$\frac{\partial v}{\partial y}=0$$

Momentum Equation

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=g\beta\left(C-C_\infty\right)+\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+\frac{\mu_e H_0}{\rho}\left(\frac{\partial H_x}{\partial y}\right)-\frac{\upsilon}{K}u$$

Magnetic Induction Equation

$$\frac{\partial H_x}{\partial t}+v\frac{\partial H_x}{\partial y}=H_0\frac{\partial u}{\partial y}+\frac{1}{\sigma\mu_e}\left(\frac{\partial^2 H_x}{\partial y^2}\right)$$

Concentration Equation

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial y^2}\right)$$

and the corresponding initial and boundary conditions are as follows,

$t\le 0,\ u=0,\ H_x=0,\ \ C\to C_\infty$ everywhere

$t>0,\ u=0,\ H_x=H_w,\ C=C_w$ at $y\to 0$

$u=0,\ H_x\to 0,\ \ C\to C_\infty$ as $y\to\infty$

where *x* & *y* are cartesian coordinates, *u* & *v* are velocity components, *g* is the local acceleration due to gravity, β is the concentration expansion coefficient, υ is the kinematic viscosity, K is the permeability of porous medium, μ_e is the magnetic permeability, ρ is the density of the fluid, H_0 is the constant induced magnetic field, H_x be the *x*-component induced magnetic field, σ is the electrical conductivity, H_w is the induced magnetic field near at plate, D_m is the coefficient of mass diffusivity.

The continuity equation gives v = constant $=-V_0$ (say), where the negative values indicate the constant suction velocity of fluid. In order to establish a non-dimensional model of this problem, it is required to make the above equations dimensionless. In this case, the dimensionless quantities are introduced as follows;

$$Y=\frac{yV_0}{\upsilon},\ U=\frac{u}{V_0},\ \ \tau=\frac{tV_0^2}{\upsilon},\ \ \bar{H}=\sqrt{\frac{\mu_e}{\rho}}\frac{H_x}{V_0},\ \bar{C}=\frac{C-C_\infty}{C_w-C_\infty}$$

After simplification, we have obtained the following nonlinear coupled partial differential equations interms of dimensionless variables,

Non-dimensional Momentum Equation

$$\frac{\partial U}{\partial\tau}-\frac{\partial U}{\partial Y}=G_r\bar{C}+\frac{\partial^2 U}{\partial Y^2}+M\frac{\partial\bar{H}}{\partial Y}-\gamma U$$

Non-dimensional Magnetic Induction Equation

$$\frac{\partial\bar{H}}{\partial\tau}-\frac{\partial\bar{H}}{\partial Y}=M\frac{\partial U}{\partial Y}+\frac{1}{P_m}\frac{\partial^2\bar{H}}{\partial Y^2}$$

Non-dimensional Concentration Equation

$$\frac{\partial\bar{C}}{\partial\tau}-\frac{\partial\bar{C}}{\partial Y}=\frac{1}{S_c}\frac{\partial^2\bar{C}}{\partial Y^2}$$

and the associated initial with boundary conditions become,

$\tau\le 0,\ U=0,\ \bar{H}=0,\ \bar{C}=0,$ everywhere

$\tau>0,\ U=0,\ \bar{H}=1,\ \bar{C}=1$ at $Y=0$

$U=0,\ \bar{H}\to 0,\ \bar{C}\to 0$ as $Y\to\infty$

where τ represents the dimensionless time, Y is the dimensionless cartesian co-ordinate, U is the dimensionless velocity component, $\bar{H}$ is the dimensionless induced magnetic field, $\bar{C}$ be the dimensionless species concentration as well as the non-dimensional parameters are also given below;

$G_r=\dfrac{\upsilon g\beta\left(C-C_\infty\right)}{V_0^3}=$ Modified Grashof number,

$M=\dfrac{H_0}{V_0}\sqrt{\dfrac{\mu_e}{\rho}}=$ Magnetic force number,

$P_m=\upsilon\sigma\mu_e=$ Magnetic diffusivity number,

$S_c=\dfrac{\upsilon}{D_m}=$ Schmidt number and

$\gamma=\dfrac{\upsilon^2}{KV_0^2}=$ Permeability number of porous medium.

3. Shear Stress, Current Density and Sherwood Number

Now we are attempt to discuss about the quantities of chief physical interest as skin friction coefficient, current density and mass transfer rate. The effects of various parameters on the shear stress near at the plate have been observed from the velocity field. The shear stress is mathematically defined by the following relations,

$\tau=\left(\dfrac{\partial U}{\partial Y}\right)_{Y=0}$ which is proportional to $\left(\dfrac{\partial U}{\partial Y}\right)_{Y=0}$ where μ denotes the coefficient of viscosity.

The effects of various parameters on the current density have been investigated by the values of induced magnetic field. The following equation represents the current density at the plate,

$$J = \mu\left(-\frac{\partial \bar{H}}{\partial Y}\right)_{Y=0} \text{ which is proportional to } \left(-\frac{\partial \bar{H}}{\partial Y}\right)_{Y=0}.$$

Also from the concentration field, the effects of different parameters on the Sherwood number have been analyzed here. The Sherwood number is defined as follows,

$$S_h = \mu\left(-\frac{\partial \bar{C}}{\partial Y}\right)_{Y=0} \text{ which is proportional to } \left(-\frac{\partial \bar{C}}{\partial Y}\right)_{Y=0}.$$

4. Numerical Solutions

In order to obtain a numerical solution of the non-dimensional system of coupled non-linear partial differential equations with the associated initial and boundary conditions, finite difference method has been used in this section. In this case it is required to obtain the finite difference equations so the region of flow within the boundary layer is divided into many grid or mesh of lines parallel to the plate where *Y*-axis is normal to the plate. In this study, it is considered that the length of the boundary layer thickness $Y_{\max}(=20)$ which as corresponding to $Y \to \infty$ i.e. Y varies from 0 to 20 within the boundary layer. It is also assumed that the number of grid is $n = 100$ in the normal direction of plate which is drawn in the following Fig. 2.

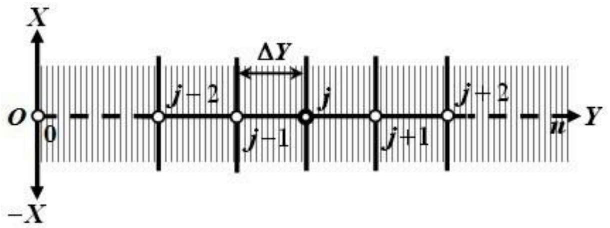

Fig. 2 Finite difference space grid

Under the above consideration, we have the constant mesh size along normal direction of plate, $\Delta Y = 0.2(0 \le Y \le 20)$ with a small time-step $\Delta\tau = 0.01$.

Let U', $\bar{H}'$, $\bar{C}'$ represent the values of U, $\bar{H}$, $\bar{C}$ at the end of a time-step respectively. The following appropriate set of finite difference equations are obtained by the use of well known finite difference approximations on the mathematical model.

Finite Difference Momentum Equation

$$\frac{U'_j - U_j}{\Delta\tau} - \frac{U_{j+1} - U_j}{\Delta Y} = G_r \bar{C}'_j + \frac{U_{j+1} - 2U_j + U_{j-1}}{(\Delta Y)^2} + M\frac{\bar{H}_{j+1} - \bar{H}_j}{\Delta Y} - \gamma U_j$$

Finite Difference Magnetic Induction Equation

$$\frac{\bar{H}'_j - \bar{H}_j}{\Delta\tau} - \frac{\bar{H}_{j+1} - \bar{H}_j}{\Delta Y} = M\frac{U_{j+1} - U_j}{\Delta Y} + \frac{1}{P_m}\frac{\bar{H}_{j+1} - 2\bar{H}_j + \bar{H}_{j-1}}{(\Delta Y)^2}$$

Finite Difference Concentration Equation

$$\frac{\bar{C}'_j - \bar{C}_j}{\Delta\tau} - \frac{\bar{C}_{j+1} - \bar{C}_j}{\Delta Y} = \frac{1}{S_c}\frac{\bar{C}_{j+1} - 2\bar{C}_j + \bar{C}_{j-1}}{(\Delta Y)^2}$$

As well as the initial and boundary conditions with the finite difference scheme are as follows,

$$U_j^0 = 0,\ \bar{H}_j^0 = 0,\ \bar{C}_j^0 = 0$$
$$U_0^n = 0,\ \bar{H}_0^n = 1,\ \bar{C}_0^n = 1$$
$$U_L^n = 0,\ \bar{H}_L^n = 0,\ \bar{C}_L^n = 0 \quad \text{where } L \to \infty$$

here the subscripts j designate the grid position on *y*-direction and the superscript *n* represents a value of time i.e. $\tau = n\Delta\tau$ where $n = 0,1,2,.....$. From the initial condition, the values of U, $\bar{H}$, $\bar{C}'$ are known at the initial time. During any one time-step, the new concentration, the new velocity and the new induced magnetic field at all interior nodal points may be computed by successive applications of the above finite difference equations. This process is repeated in time and provided the time-step is sufficiently small, hence U, $\bar{H}$, $\bar{C}$ should eventually converge to values which approximate the steady-state solution of the model. Finally, the numerical values of shear stress, current density and Sherwood number are evaluated by the use of Five-point differential formula.

6. Results and Discussion

To discuss the effects of parameters on flow variables, the numerical solutions of the problem are obtained by the finite difference method. A computer programming language software ***Compaq Visual Fortran*** is used to develop a Fortran program for finite difference technique. For discussing the physical aspects of the problem, we have computed the steady state values of fluid velocity, induced magnetic field and species concentration within the boundary layer for different values of magnetic force number, magnetic diffusivity number, modified Grashof number, Schmidt number and permeability number for porous medium. It is observed from this computation that the values of flow variables show little changes after the 20 time and a negligible change is observed after the 40 time. Hence, the obtained values at the time 80 are essentially steady state solutions. In this study, the fluids are chosen helium $(S_c = 0.3)$ for lighter particles as well as water-vapor $(S_c = 0.6)$ and carbondioxide $(S_c = 1.0)$ for heavier particles at $25^0 C$ temperature and one atmospheric pressure. The values of other associated parameters are also chosen arbitrarily. Finally, the obtained numerical values of flow variables versus the *Y*-directional length coordinate are illustrated in Figs. 3-12 with the help of data visualization software ***TECPLOT***.

The effects of various parameters on fluid velocity have been shown in Figs. 3-6. It is observed from Fig. 3 that the velocity increases with the rise of modified Grashof number. The Fig. 4 shows that the velocity decreases in case of strong permeability number.

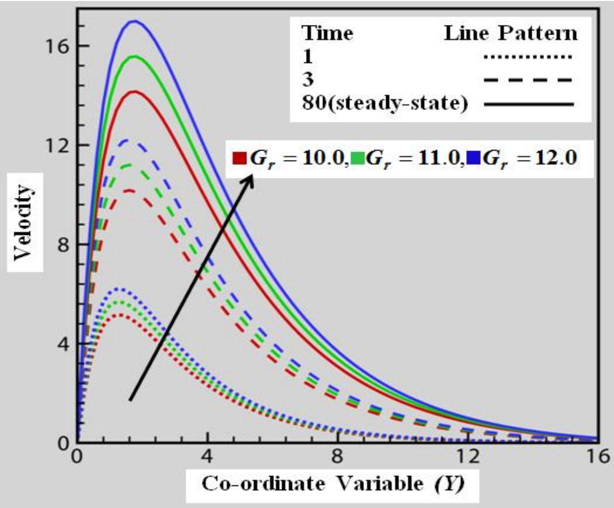

Fig. 3 Transient velocity profiles for different values of G_r with $M = 0.1, \gamma = 0.1, S_c = 0.3, P_m = 1.0$

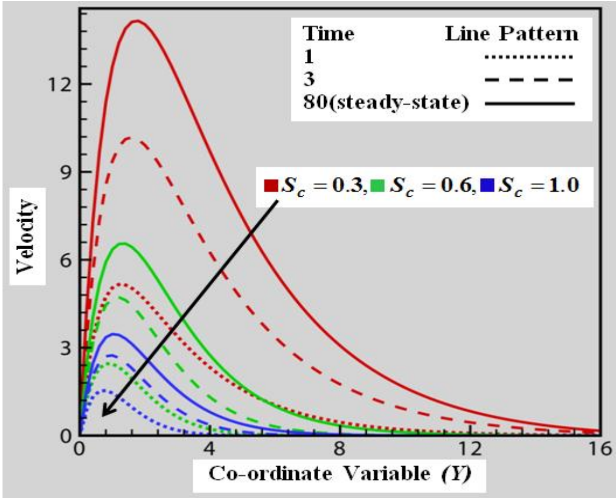

Fig. 6 Transient velocity profiles for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1.0, M = 0.1$

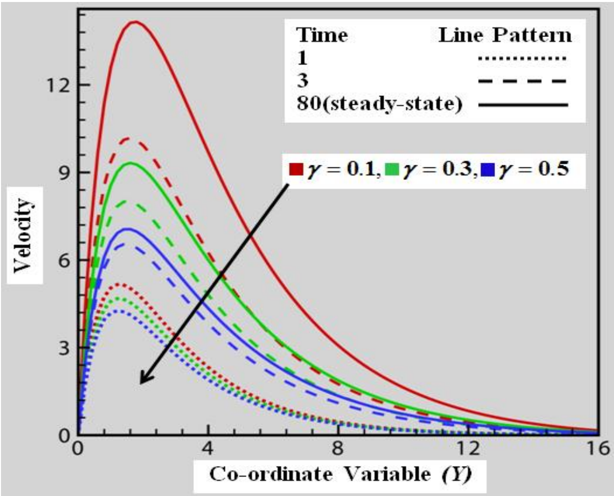

Fig. 4 Transient velocity profiles for different values of γ with $M = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

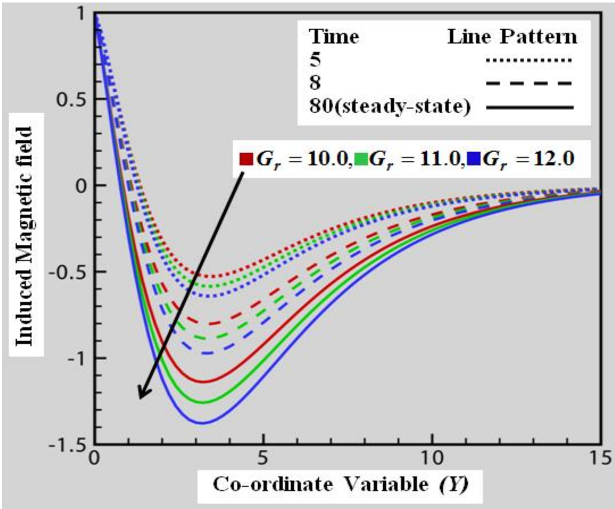

Fig. 7 Transient induced magnetic field for different values of G_r with $M = 0.1, \gamma = 0.1, S_c = 0.3, P_m = 1.0$

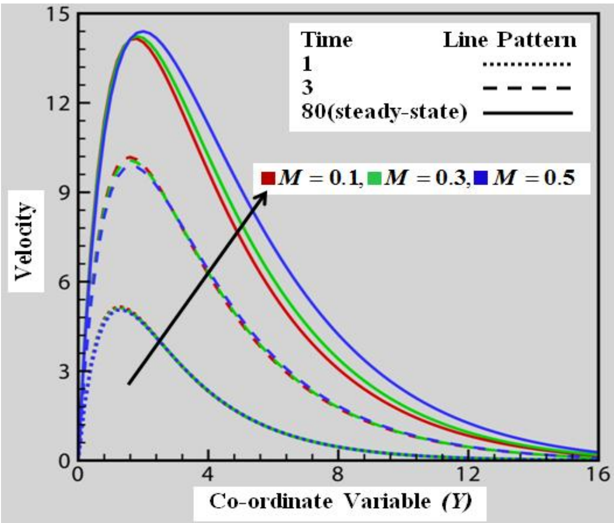

Fig. 5 Transient velocity profiles for different values of M with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

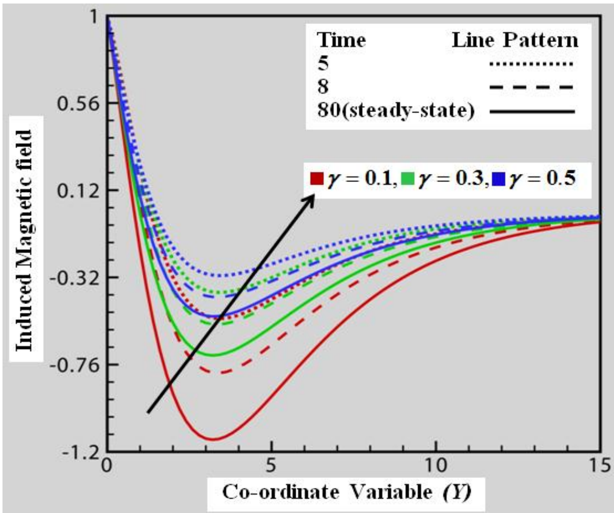

Fig. 8 Transient induced magnetic field for different values of γ with $M = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

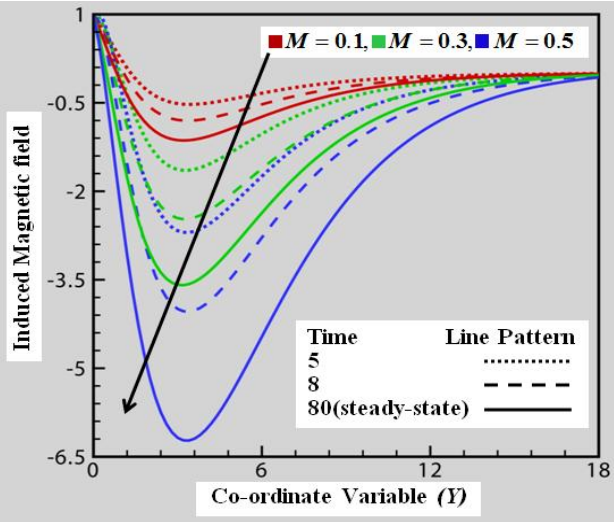

Fig. 9 Transient induced magnetic field for different values of M with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

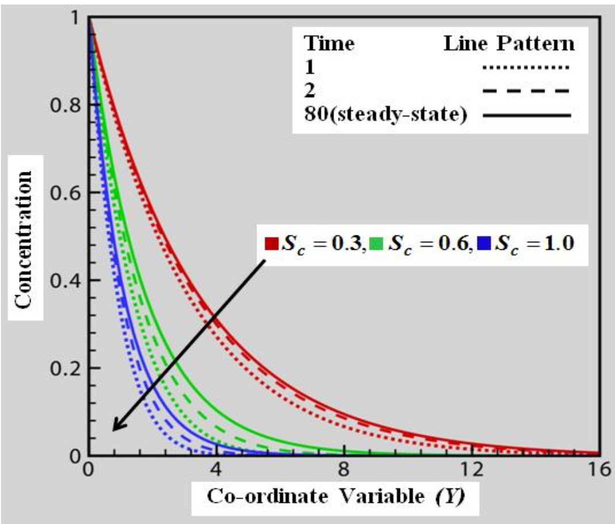

Fig. 12 Transient concentration profiles for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1.0, M = 0.1$

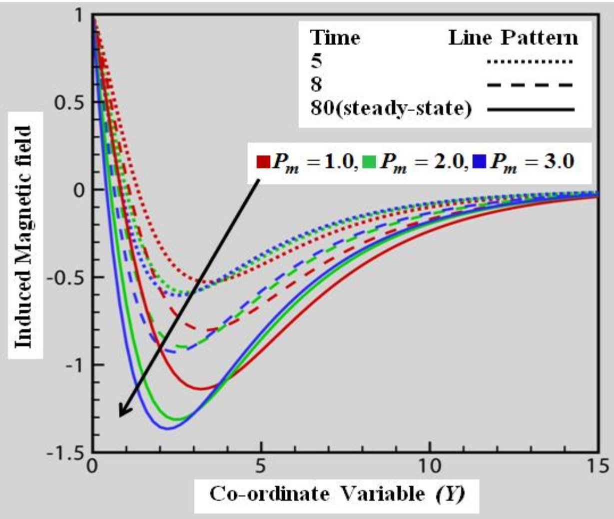

Fig. 10 Transient induced magnetic field for different values of P_m with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, M = 0.1$

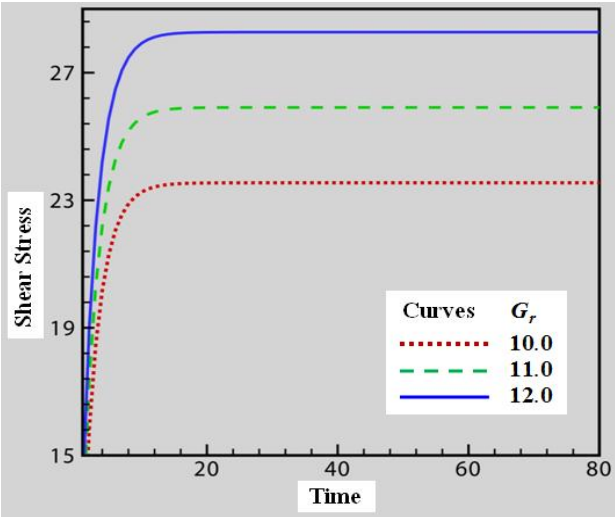

Fig. 13 Shear stress for different values of G_r with $M = 0.1, \gamma = 0.1, S_c = 0.3, P_m = 1.0$

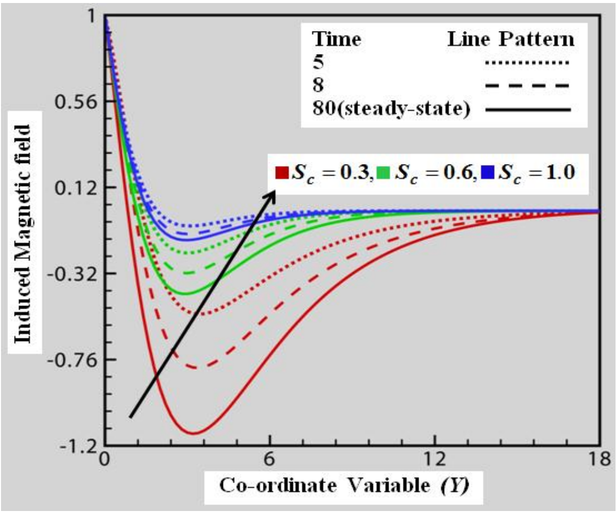

Fig. 11 Transient induced magnetic field for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1.0, M = 0.1$

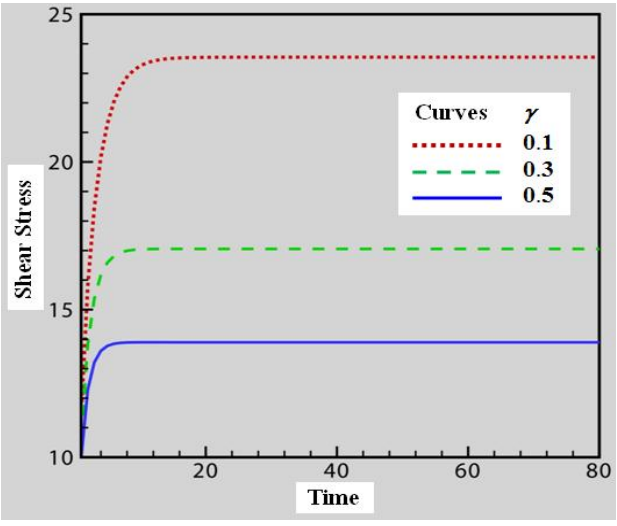

Fig. 14 Shear stress for different values of γ with $M = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

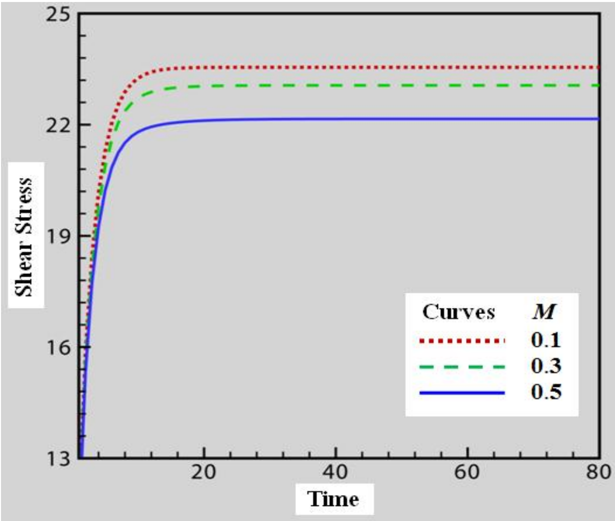

Fig. 15 Shear stress for different values of M with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

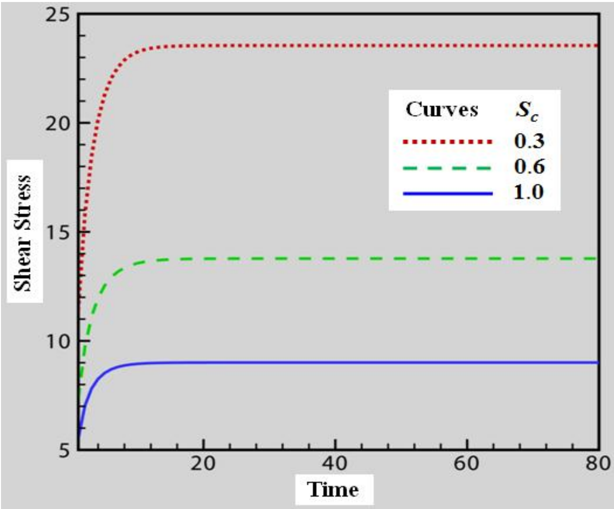

Fig. 16 Shear stress for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1, M = 0.1$

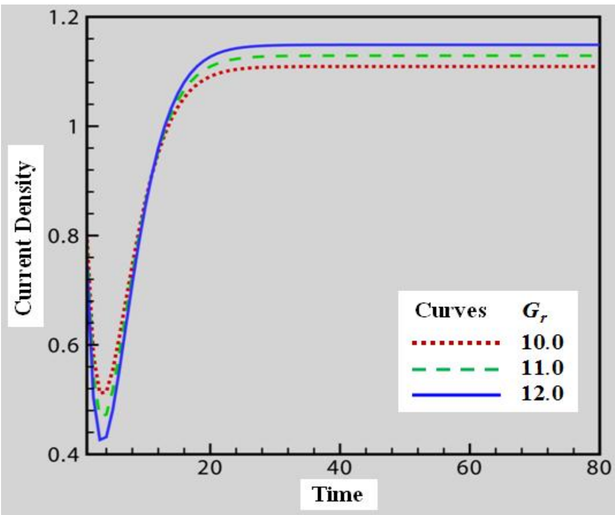

Fig. 17 Current density for different values of G_r with $M = 0.1, \gamma = 0.1, S_c = 0.3, P_m = 1.0$

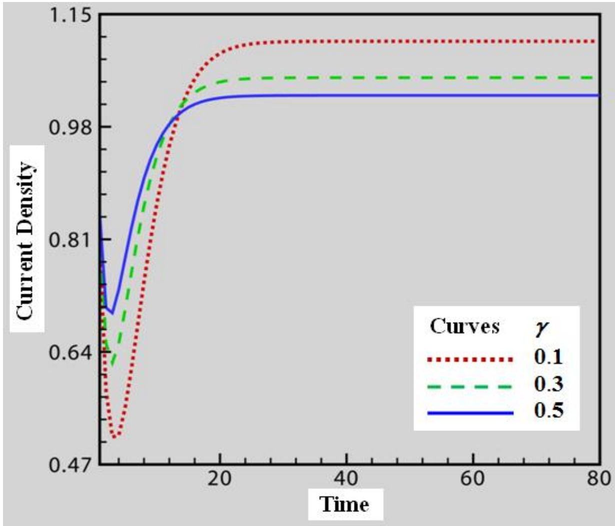

Fig. 18 Current density for different values of γ with $M = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

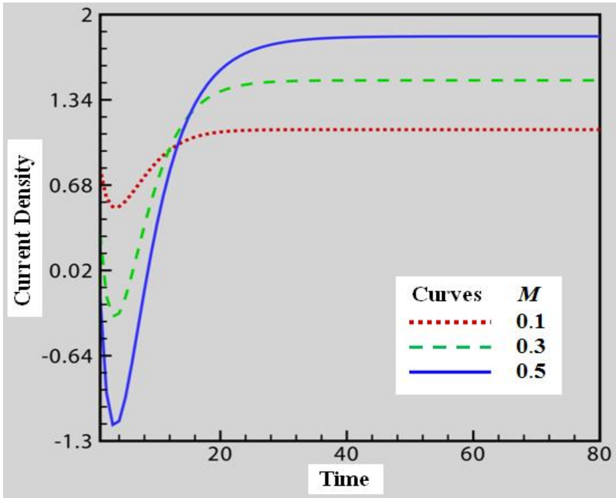

Fig. 19 Current density for different values of M with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, P_m = 1.0$

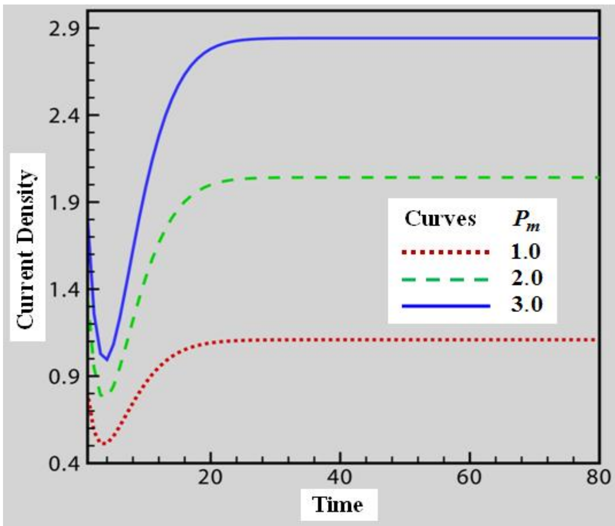

Fig. 20 Current density for different values of P_m with $\gamma = 0.1, G_r = 10.0, S_c = 0.3, M = 0.1$

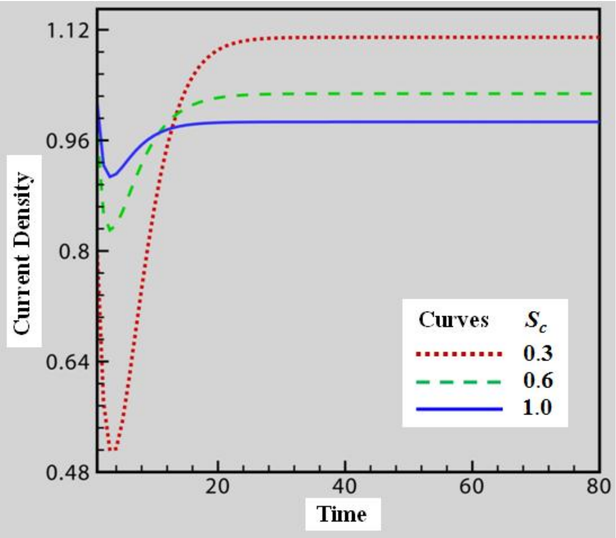

Fig. 21 Current density for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1.0, M = 0.1$

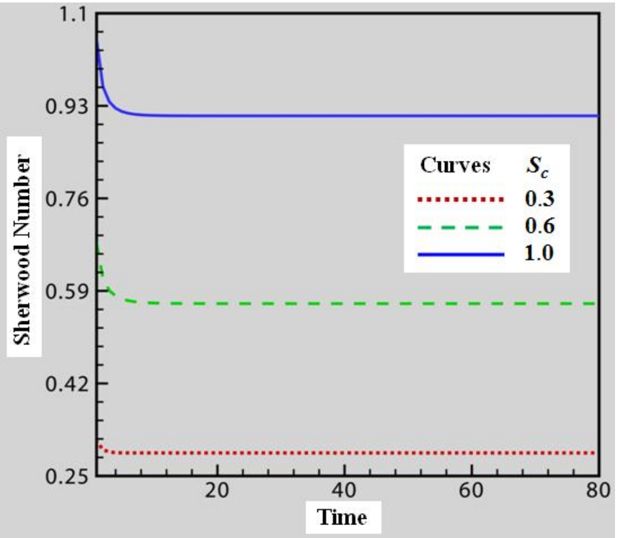

Fig. 22 Sherwood number for different values of S_c with $\gamma = 0.1, G_r = 10.0, P_m = 1.0, M = 0.1$

An increasing effect of magnetic force number on velocity at steady-state period is observed from the Fig. 5. It is noted from Fig. 6 that the fluid velocity strongly decreases for the use of high concentric fluid.

The curves of induced magnetic field are drawn in Figs. 7-11 for the values of flow parameters. In Fig. 7, we see that the induced magnetic field decreases with the increase of G_r. The Fig. 8 shows that the induced magnetic field gradually increases for the rise of Permeability number. A strong decreasing effect of magnetic force number on the induced magnetic field is observed from Fig. 9. We see in Fig. 10, the induced magnetic field strongly decreases near the plate but slowly increases far away from the plate in case of strong P_m. It is observed from Fig. 11 that the induced magnetic field is increasingly affected by the Schmidt number.

An increasing effect of Schmidt number on the species concentration is noted from Fig. 12. It is also observed from Figs. 3-12 that the fluid velocity and species concentration increase while the induced magnetic field decreases with the change of time.

Now we are attempt to discuss about the behavior of quantities of chief physical interest as shear stress, current density and Sherwood number for different values of associated parameters. For this purpose, the numerical values of the above quantities are computed and the obtained values versus non-dimensional time are plotted in Figs. 13-22.

The profiles of shear stress are displayed in Figs. 13-16 for the different values of flow parameters. It is observed from these figures that the shear stress increases only for the modified Grashof number while it decreases with the increase of permeability number or magnetic force number or Schmidt number. The curves of current density are shown in Figs. 17-21 and we see in these figures, before 20 time the current density gradually increases in case of strong permeability number or magnetic force number or Schmidt number while it decreases for the increasing values of modified Grashof number but after 20 time the reverse effects of same parameters on current density are noted from these figures. It is also observed from Fig. 20 that the current density is increasingly affected by magnetic diffusivity number at any time. Finally, the Fig. 22 shows that the Sherwood number strongly increases for the increasing values of Schmidt number.

Conclusions

In this work, a natural convective mass transfer transient flow of a viscous incompressible fluid through a porous medium has been investigated in the presence of a strong magnetic field that induced another magnetic field. The mathematical model of flow has been solved numerically by finite difference method. Finally, the results are discussed for different values of flow parameters and the important findings that obtained from the graphical representation of the results are listed below;

1) The fluid velocity is higher for lighter particles than heaver particles.
2) The induced magnetic field decreases in case of strong magnetic force number or magnetic diffusivity number. Particularly, the induced magnetic field is greater for heaver particles than lighter particles.
3) The concentration level of fluid is greater for carbondioxide than helium.

These findings may be useful in the processes of separation, in chemical engineering as well as in many industrial applications as the drying of solid materials, distillation, extraction and absorption.

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